Summary of Research Activities Summer 2003

Justin Lambright
Joseph Berg
Geneva College
3200 College Avenue
Beaver Falls, PA 15010

Joe and I arrived at Geneva College on Memorial day. Dr. van de Sande gave us some reading material to prepare us for the summer. We were given *The Feynman Lectures on Physics* by Feynman, Leighton, and Sands along with *Spacetime Physics* by Taylor and Wheeler to read about relativity. We were given seperate books to read about particles; Joe was given *Introduction to High Energy Physics* by Perkins, and I was given *Quarks and Leptons: An Introductory Course in Modern Particle Physics* by Halzen and Martin. We were instructed to do the questions from the first chapter. The first chapter of the books introduced a summary of the particles that make up our world and the forces that act on them. It introduced the idea of color, flavor, and Quantum Chromodynamics, as well as natural units in which $\hbar$ and the speed of light both equal one. We spent the next week being lectured about the Standard Model, the four basic forces (introducing the strong force, which relates to color, along with the weak force), particles (eg. gauge bosons, fermions, quarks, leptons, mesons, and baryons), and Light Front Coordinates. Each day Joe and I worked on a few examples, such as looking at the Lorentz boost in Light Front Coordinates. Other examples we did were from *Introduction to Quantum Mechanics* by David J. Griffiths, such as looking at the Time Independent Schrödinger Equation in Light Front Coordinates. We also looked at fermions a little closer and the Pauli Exclusion Principle. We reviewed the creation and annihilation notation along with commutators, introducing the anti-commutator for quarks and antiquarks. Examples with Angular Momentum ladder operators were worked out to review some principles we had discussed in Quantum Mechanics class the previous semester. A few examples looked at the normalization for states with several link fields, four-momentum in light front coordinates, the Pauli spin matrices, and the Levi-Civita symbol (ϵ_{ijk}). We were introduced to the transverse lattice approach to QCD we would be using when we began the project.

In preparation for our work on the baryons Joe and I were instructed to recreate some of the work done by Jonathon Bratt and Beth Watson on mesons in the summer of 2001. This was to help us learn to use *Mathematica*, as well as become familiar with the material. Since mesons are simpler than baryons, understanding them would provide us with a foundation for the work with baryons. Dr. van de Sande divided the work, giving each of us a different part of the code to write. Joe wrote functions to create a basis of states; while I wrote functions which manipulated the states, such as rotating and flipping them about differnt axes. I then wrote functions to allow Joe to create a reduced basis using multiplets from the website `http://www.geneva.edu/~bvds/three`. We then looked at the inner product and how it worked, eventually adding functions to our code. Our code was then checked to make sure we got the same results as the old meson code.

After completing the code for the mesons, we started work on baryons. Our first task was to find S_x, S_y, and S_z for $S = \frac{3}{2}$. As an exercise in understanding intrinsic angular momentum we calculated these three matrices for values of $S = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}$. Once we had S_x, S_y, and S_z we could then calcuate R (90° rotation on lattice), R_1^2 (180° rotation about the x^1 axis), and R_2^2 (180° rotation about the x^2 axis). We then used these to calculate the values used in the multiplets for the baryons. In attempting to calculate the 180° rotations we found problems with parity, which eventually led us to remove the reflections in the x^1 and x^2 directions from the multiplets. Later, Dr. van de Sande eventually four multiplets that included the reflections. We worked out the coefficients for the identity, R, R^2, and R^3 for the multiplets (see `http://www.geneva.edu/~bvds/baryon`). Joe wrote a summary of these calculations using Latex. Once we finished these calculations, Joe and I both wrote functions to create a basis of states, as well as a reduced basis and inner products for baryons. By doing this independently we were able to check our work with each other. After completing the basis functions, Joe worked on modifying the meson interactions to work for the baryons, while I started to write functions to create the Hamiltonian matrix using the interactions from Joe's code. Baryon states (for example

$$|\psi\rangle = \epsilon_{ijk}(b^\dagger a^\dagger a^\dagger)_i(b^\dagger a^\dagger)_j(b^\dagger a^\dagger)_k|0\rangle \ ,$$

where $b^\dagger$ creates a quark, $a^\dagger$ creates a gluon, and $|0\rangle$ is the vacuum) are a little more complicated than meson states (for example

$$|\psi\rangle = b^\dagger a^\dagger a^\dagger \dots d^\dagger|0\rangle \ ,$$

where $d^\dagger$ creates an antiquark). However, the baryon interactions that occur within one leg (one quark and its chain of gluons) were the same as those acting on mesons and easily copied over.

The next step was to work out the determinant operator, four quark interaction, and the interactions between legs. We looked up the operators in *Transverse lattice calculation of the pion light-cone wavefunction* and *Glueballs on a transverse lattice* both by Simon Dalley and Brett van de Sande. We worked out by hand which interactions were leading order in N (the number of colors), which states they acted on, and what they looked like once sandwiched between two states. The determinant and four quark interactions were written in my code, while Joe wrote a function for the between leg interactions. Once an interaction was finished we would test our code with Dr. van de Sande's code. Due to the large calculations being done, I had to rewrite some code to make it run faster. After finding many mistakes and a good deal of debugging, we managed to get it all running with agreement by the end of July. We then went home for a couple weeks, while Dr. van de Sande went to England.

We returned about a month later for one last week before classes resumed. We then worked on fixing a problem with the meson calculations done by Jonathon Bratt and Beth Watson. Dr. van de Sande told us another operator needed to be added to the Hamiltonian operator. The new operator, $(\bar{\psi}\gamma_5\psi)^2$, will hopefully fix some of the problems associated with chiral symmetry breaking. This chiral four fermion operator was worked out by hand, then added to Joe's copy of Beth Watson's code. After three days, Dr. van de Sande and we had agreement. Thanks to a summer of working with *Mathematica* and Dr. van de Sande, we had learned enough to finish this final task fairly quickly.