Transverse Lattice QCD

Abstract

The transverse lattice is a non-perturbative approach for solving the bound state problem in the theory of Quantum Chromodynamics. The goal of this work is to calculate various properties of the hadrons, such as structure functions, which are of great experimental interest but are difficult or impossible to calculate using other approaches.

Introduction

There are four fundamental forces which seem to describe all known phenomena: gravity, electromagnetism, the "weak" force, and the "strong" force. We are all familiar with the forces of gravity and electromagnetism while the latter two can only be seen at sub-atomic distances. During the last century, physicists have discovered a menagerie of subatomic particles: protons, neutrons, pions, kaons, and thousands of their cousins. We believe that these particle, called "hadrons," are composed of either two or three "quarks." It is the strong force that describes how these quarks are bound together to make a proton, neutron, pion, or whatever.

Quantum Chromodynamics, or QCD for short, is the 'scientific' name of the theory that describes how quarks interact with each other via the strong force. The theory of QCD was developed during the early 1970's, and it received convincing experimental confirmation from experiments performed at the Stanford Linear Accelerator (SLAC) during that time.

On the experimental side, QCD is well understood. A myriad of masses, decay rates, and other properties of a variety of hadrons have been studied in detail. In particular, the behavior of these particles in high-energy scattering experiments has been measured to great precision. The results of these experiments are parametrized by the so-called "structure functions."

On the theoretical side, however, QCD has proven to be quite a challenge. Indeed, our theoretical understanding of the theory lags far behind experiment. Much of the current theoretical effort to understand QCD has focused on large-scale numerical calculations: Euclidean lattice Monte Carlo (ELMC). This approach has been somewhat successful in predicting masses and some other quantities. However, the present situation is far from ideal:

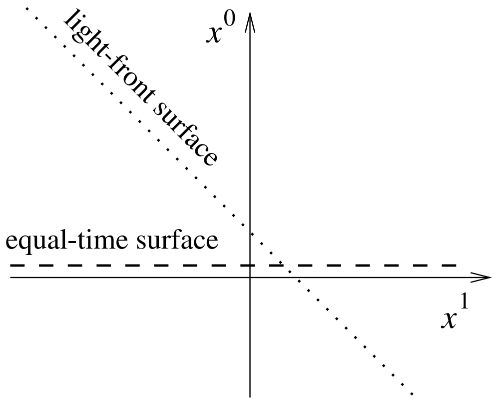

Figure 1: Conventionally, one quantizes a field theory on a surface of constant x^0 (time). Instead, we quantize on a surface of constant $x^+ = (x^0 + x^2)/\sqrt{2}$ (null plane).

- The Euclidean lattice approach is, to date, the only successful quantitative approach. There are few independent checks on the correctness of the method.
- The method, consisting of large-scale numerical calculations, can act as a sort of 'black-box.' Even if one does get the right answer, the *how* and *why* can remain obscure.
- There are physical quantities of great experimental interest, such as the structure functions mentioned above, which are not easily calculated using Euclidean lattice methods.

In conclusion, it is essential to develop alternative approaches to the study of QCD. This is the principle motivation for our work on the transverse lattice.

A light-front Hamiltonian approach

The construction of the transverse lattice [1, 2] is as follows: In the two "transverse" space dimensions a lattice is employed while light-front Hamiltonian techniques are used in the remaining two space-time dimensions. We introduce "link fields" that lie on each link of the lattice in the two transverse directions x^1 and x^2. Normally, one quantizes a field theory on a surface of constant x^0. Instead, as first suggested by Dirac [3], we quantize a field theory on a surface of constant $x^+ = (x^0 + x^2)/\sqrt{2}$; see Fig. 1. This is a more natural way to quantize a highly relativistic theory (like QCD).

Calculationally, we use a Hamiltonian approach: we create a basis of states consisting of link fields joined together to form a string. Next, we construct the Hamiltonian matrix in this basis. Finally, we calculate the lowest eigenvalues and eigenvectors of the Hamiltonian matrix; these are the masses and wavefuntions of the hadrons. The wavefunctions contain all the information needed to find the structure functions and other quantities of experimental interest [4]. Further information can be found in Ref. [8] and references therein.

Current status

My involvement in this line of investigation began approximately five years ago. During this time I have collaborated with Simon Dalley (Cambridge University) [5, 6, 7, 8] and with Matthias Burkardt (U. of New Mexico, Las Cruces) [9]. During this time, we have studied a series of increasingly difficult — and more realistic — problems starting with model field theories in two space-time dimensions and culminating with an independent, first-principles calculation of the glueball spectrum this year.

What are glueballs? The quantum mechanical particles that are responsible for the strong force are called "gluons," just as photons are the particles associated with electromagnetism. It has been hypothesized that one can construct a particle entirely out of these gluons — the glueball. There are several experimental candidates for such a particle at a mass of 1.5 to 2 times the mass of the proton. Fig. 2 shows some of our results.

In order to perform these calculations, we have taken the limit of an infinite number of colors N_c (this is just the number of different charges in the theory). This limit was taken in order to make our numerical calculations easier. For instance, one can, in this limit, ignore quarks. However, in the real world, $N_c = 3$ and quarks are important. Nevertheless, we see that our results, Fig. 2, are already quite close to the $N_c = 3$ case.

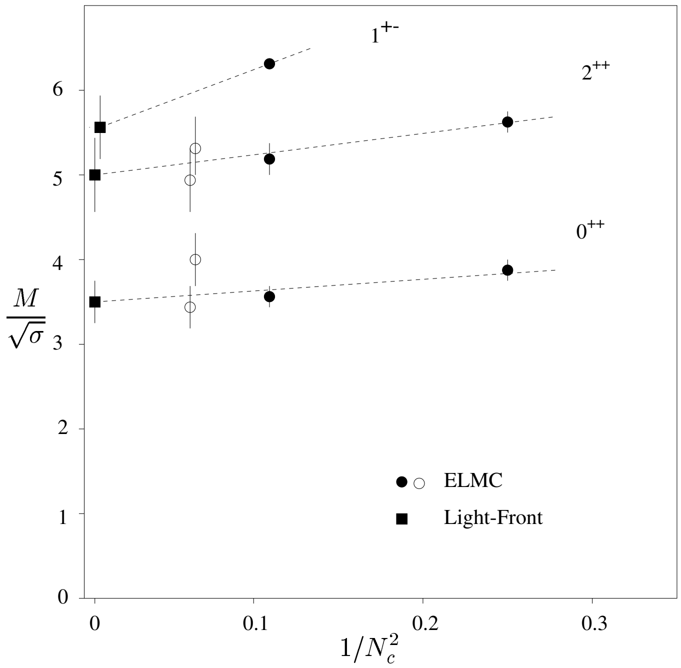

Figure 2: The glueball masses (in units of the string tension σ) as a function of N_c. ELMC are shown for $N_c = 2$, 3, and 4 [10, 11, 12, 13]; our results are shown at $N_c = \infty$. The dotted lines correspond to leading linear dependence on $1/N_c^2$.

The next step

The success of this work to date has encouraged us to take the next steps towards the world of experiment: the introduction of a finite number of colors N_c along with the the inclusion of quarks in the theory. While the inclusion of a finite number of colors N_c is more of a computational problem, the inclusion of quarks contains some conceptual difficulties that have not yet been resolved.

Thus, we will probably have to take a step backwards and study some toy models before tackling "the real thing." The models that we have in mind consists of a limited number of quarks and gluons: for instance two quarks or two quarks plus a gluon. Our current plan is to gradually add more particles as we resolve the various problems that will arise.

Computational issues

Any attempt to include Fermions in the transverse lattice will contain significant computational challenges. To date, we have included as many as 50,000 states in our basis. Thus, we have been diagonalizing 50,000 by 50,000 Hamiltonian matrices. Since these matrices are sparse (many elements are zero), we can use Lanczos techniques to find the lowest eigenvalues and eigenvectors. This takes a few minutes on a PC. However, when we include Fermions, the number of states in the basis will certainly be an order of magnitude larger and both the construction and the diagonalization of the Hamiltonian matrix will become very time consuming.

Thus, we need to develop a parallelized version of our computer code suitable for a modern parallel computer like the IBM SP2 or the CRAY T3E. (Although we have extensive experience with running our code on such machines, the calculations have not been, to date, truely parallel: each individual matrix diagonalization is performed on a single processor.) The computer code that generates the Hamiltonian matrix can easily be parallelized. What is more difficult is the diagonalization of this matrix. There are several parallel algorithms available for the diagonalization of a general matrix [14] but it is not clear that they can be applied in an efficient manner to our situation (our matrices are symmetric).

Student Involvement

Typically, one does not think of quantum field theory as a subject area where undergraduate students can become involved in research. To some extent, this is true for the for our project. On the other hand, our approach is to the problem essentially Hamiltonian in nature and a lot of what we do (such as eigenvalues and eigenvectors) can be comprehended by a student who has studied quantum mechanics. Numerical solution of some of the simple models, such as a two-quark model [15], can be tackled by a motivated undergraduate with a background in quantum mechanics.

A more obvious place where students can be involved is in the development and/or testing of parallel computer codes for sparse matrix diagonalization. This is work involves state-of-the art issues in parallel computing but is one place where a undergraduates can contribute. Why? The basic mathematical operation in the Lanczos algorithm is the matrix-vector product, something familiar to an undergraduate who has taken a course in numerical methods.

Thus, I hope to find two students to work on the project: one to conduct a more physics-oriented study of models containing Fermions and the other to work on the computational issues of diagonalizing matrices on parallel computers. Currently, I am teaching a junior level course on numerical methods at Geneva college and have identified a number of students in that course who would be qualified for either part of this project.

Impact

If this work is successful, it would represent a major breakthrough in our understanding of the theory of QCD. We would be able to, from first principles, calculate for the first time many of the quantities that are of great experimental interest.

On the computational side, this work may lead to the development of an efficient parallel algorithm for the diagonalization of sparse symmetric matrices. This would be of interest to many branches of physics as well as other fields of science and engineering.

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