■ Analyze data(new style March 1998)

Raw data from C program. The following gives the various parameters and truncations used in the numerical calculation.

```
In[4]:= data[[1]]

Out[4]= {******* Eigenvalues for the 2+1 transverse lattice  *******,
         Couplings:  m^2, g^2 N/a, la_1/a, la_2/a, la_3/a, tau,
                      0      1       2       3       4      5,
         Use chi^2 fit with 12 criteria, and tolerance 0.001.,
         Overall scale determined by best chi^2 fit.,
         2 parity doublets with fractional errors 2 0.5.,
         Spectrum for P_perp a = ( 0)  ( 0.25) ,
           using (# states, o, multiplet, c^2 error for each) =,
           (4, 1 & -1, 1 & 2, 0.5 2 0.5 0.5),   (4, 1 & -1, 2 & 1, 0.5 0.5 0.5 0.5).,
         Spectra extrapolated using (K,p) = (18/2,6)  (18/2,8) ,
           (20/2,6)  (20/2,8)  (24/2,6)  (32/2,6) .,
         Winding potential using (n,K,p) = ( 2,20/2,4)  ( 2,20/2,6) ,
           ( 2,24/2,4)  ( 2,28/2,4)  ( 3,21/2,5)  ( 3,21/2,7)  ( 3,23/2,5) ,
           ( 3,27/2,5)  ( 4,20/2,6)  ( 4,20/2,8)  ( 4,22/2,6)  ( 4,26/2,6) .,
         Heavy potential determined using (n,K,p,K_max) = ( 0,-32/2,2,4) ,
           ( 0,-32/2,4,4)  ( 0,-32/2,2,5)  ( 0,-34/2,2,4)  ( 0,-44/2,2,5) ,
           ( 0,-60/2,2,4) ,,   L = 3 4 6 (all in g^2 N/a units); with relative error 0.25.,
         Roundness determined using (n,K,p,K_max) = ( 1,-19/2,3,4) ,
           ( 1,-19/2,5,4)  ( 1,-19/2,3,5)  ( 1,-33/2,3,4)  ( 1,-33/2,3,5) ,
           ( 1,-49/2,3,4) ,,   L=3 and error 0.3 (all in g^2 N/a units).,
         p-extrapolation using (n,K,p) = ( 1,19/2,3)  ( 1,25/2,3) ,
           ( 1,39/2,3)  ( 1,19/2,5)  ( 1,25/2,5)  ( 1,23/2,7) ., Result format:,
          fit info, # steps, chi^2, p damping, and scale g^2 N/(a sigma);,
          the 6 couplings  (g^2 N/a units) and which--if any--were fit;,
          winding and longitudinal string tension fits;,
          n=1 L=3 eigenvalue and derived value (g^2 N/a units);,
          the rescaled spectrum for each P_perp*a and c^2 values.}
```

An example data point:

```
In[5]:= data[[2, -3]]

Out[5]= {2, 72, 20.9469, -1.77198, 4.52802, 0.145309, 1., -0.203166, -0.139667, 3.49633,
         -1.05437, 2, 3, 4, 5, 0.5671, -0.405581, -0.032258, 0.24682, -0.225622, 0.139246,
         0.929006, 0.926023, 18.8573, 18.8761, 0.773472, 32.9007, 32.9124, 0.481288, 37.4848,
         37.4717, -0.537372, 41.187, 41.1995, 0.513862, 24.8405, 24.8464, 0.241954,
         52.2753, 52.2799, 0.190021, 56.9835, 56.9918, 0.343691, 66.1505, 66.1528, 0.094404}
```

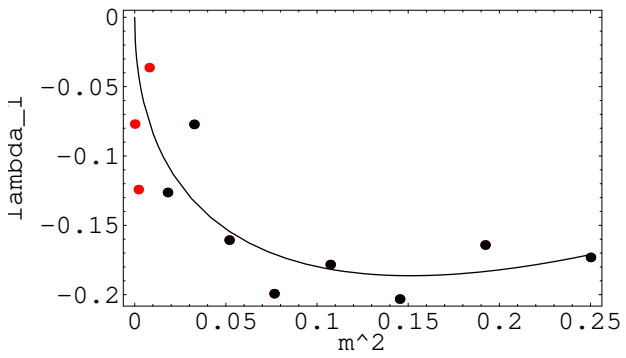
Eigenvalue of ground state

```
In[6]:= ground = Map[(#[[{6, -24 / 2}]])&, data[[2]]]

Out[6]= {{0, 19.3962}, {0.001968, 17.992}, {0.007919, 18.3082}, {0.018006, 20.7177},
         {0.032492, 20.6811}, {0.051777, 23.7316}, {0.076429, 23.6472}, {0.10724, 23.2084},
         {0.145309, 24.8405}, {0.192168, 25.7015}, {0.25, 26.7778}}
```

lambda_1 along trajectory: lambda_1/(g^2 N) vs m^2

```
In[7]:= la1 = Map[(#[[{6, 8}]] / #[[7]])&, Select[data[[2]], (5 < #[[24]] < 25)&]]

Out[7]= {{0.018006, -0.126321}, {0.032492, -0.077209}, {0.051777, -0.160642},
         {0.076429, -0.199297}, {0.10724, -0.178291}, {0.145309, -0.203166},
         {0.192168, -0.164193}, {0.25, -0.173124}}

In[34]:= fitting1 = Fit[Drop[Drop[la1, 0], 0], {Sqrt[x], x}, x]

Out[34]= -0.958237 √x + 1.23207 x
```

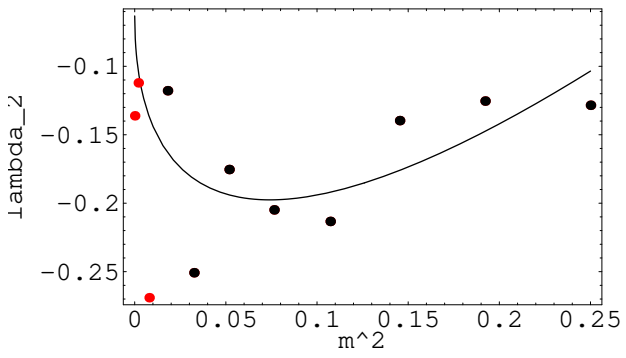
```
In[35]:= Show[Plot[fitting1, {x, 0, 1 / 4}, DisplayFunction -> Identity],
          ListPlot[Map[(#[[{6, 8}]] / #[[7]])&, data[[2]]],
           PlotStyle -> {PointSize[.02], Hue[0]}, DisplayFunction -> Identity],
          ListPlot[la1, PlotStyle -> {PointSize[.02]}, DisplayFunction -> Identity],
          DisplayFunction -> $DisplayFunction, Axes -> False, Frame -> True,
          FrameLabel -> {"m^2", "lambda_1"}]
```




```
Out[35]= - Graphics -
```

lambda_2 along trajectory: lambda_2/(g^2 N) vs m^2

```
In[10]:= la2 = Map[(#[[{6, 9}]] / #[[7]])&, Select[data[[2]], (5 < #[[24]] < 25)&]]

Out[10]= {{0.018006, -0.117837}, {0.032492, -0.250795}, {0.051777, -0.175395},
          {0.076429, -0.204872}, {0.10724, -0.213307}, {0.145309, -0.139667},
          {0.192168, -0.125382}, {0.25, -0.128447}}

In[32]:= fitting2 = Fit[Drop[Drop[la2, 0], 0], {1, Sqrt[x], x}, x]

Out[32]= -0.0631029 - 0.987651 √x + 1.81374 x

In[33]:= Show[Plot[fitting2, {x, 0, 1 / 4}, DisplayFunction -> Identity],
          ListPlot[Map[(#[[{6, 9}]] / #[[7]])&, data[[2]]],
           PlotStyle -> {PointSize[.02], Hue[0]}, DisplayFunction -> Identity],
          ListPlot[la2, PlotStyle -> {PointSize[.02]}, DisplayFunction -> Identity],
          DisplayFunction -> $DisplayFunction, Axes -> False, Frame -> True,
          FrameLabel -> {"m^2", "lambda_2"}]
```



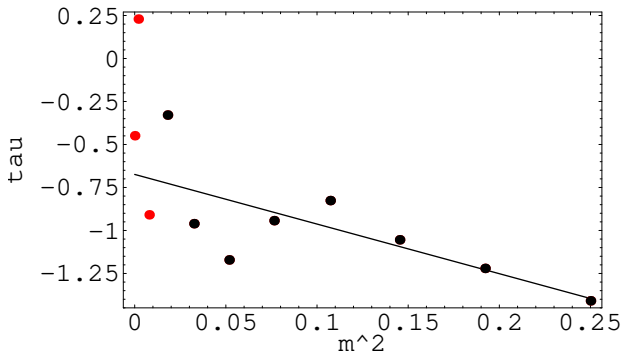

```
Out[33]= - Graphics -
```

lambda_3 along trajectory: lambda_3/(g^2 N) vs m^2

```
In[13]:= la3 = Map[(#[[{6, 10}]] / #[[7]])&, Select[data[[2]], (5 < #[[24]] < 25)&]]

Out[13]= {{0.018006, 3.50036}, {0.032492, 221.03}, {0.051777, 7.37915}, {0.076429, 6.45752},
          {0.10724, 9.40075}, {0.145309, 3.49633}, {0.192168, 2.06865}, {0.25, 2.3097}}
```

tau along trajectory: tau/(g^2 N) vs m^2

```
In[14]:= tau = Map[(#[[{6, 11}]] / #[[7]])&, Select[data[[2]], (5 < #[[24]] < 25)&]]

Out[14]= {{0.018006, -0.328865}, {0.032492, -0.960551}, {0.051777, -1.17122},
          {0.076429, -0.942923}, {0.10724, -0.826222}, {0.145309, -1.05437},
          {0.192168, -1.22054}, {0.25, -1.40905}}

In[28]:= fittingtau = Fit[Drop[Drop[tau, 0], 0], {1, x}, x]

Out[28]= -0.674107 - 2.88621 x

In[29]:= Show[Plot[fittingtau, {x, 0, 1 / 4}, DisplayFunction -> Identity],
          ListPlot[Map[(#[[{6, 11}]] / #[[7]])&, data[[2]]],
           PlotStyle -> {PointSize[.02], Hue[0]}, DisplayFunction -> Identity],
          ListPlot[tau, PlotStyle -> {PointSize[.02]}, DisplayFunction -> Identity],
          DisplayFunction -> $DisplayFunction, Axes -> False, Frame -> True,
          FrameLabel -> {"m^2", "tau"}]
```

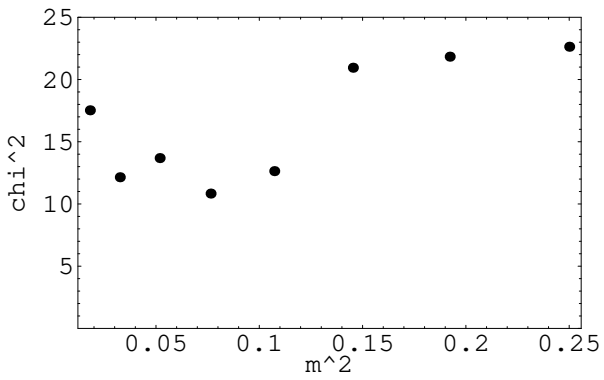



```
Out[29]= - Graphics -
```

chi^2 of fit

```
In[17]:= chi2 = Map[{#[[6]] / #[[7]], #[[3]]}&, Select[data[[2]], (5 < #[[24]] < 25)&]]

Out[17]= {{0.018006, 17.5267}, {0.032492, 12.1472}, {0.051777, 13.6865}, {0.076429, 10.8355},
          {0.10724, 12.6389}, {0.145309, 20.9469}, {0.192168, 21.8354}, {0.25, 22.6331}}
```

```
In[25]:= ListPlot[chi2, PlotRange -> {0, 25}, PlotStyle -> {PointSize[.02]}, Axes -> False,
          Frame -> True, FrameLabel -> {"m^2", "chi^2"}]
```




```
Out[25]= - Graphics -
```

Parameter for exponential damping with particle truncation: c_p vs m^2

```
In[19]:= Map[{#[[6]] / #[[7]], #[[4]]}&, Select[data[[2]], (5 < #[[24]] < 25)&]]

Out[19]= {{0.018006, -1.16803}, {0.032492, -1.07911}, {0.051777, -1.18357},
          {0.076429, -1.14551}, {0.10724, -1.4465}, {0.145309, -1.77198},
          {0.192168, -2.21382}, {0.25, -2.25724}}
```

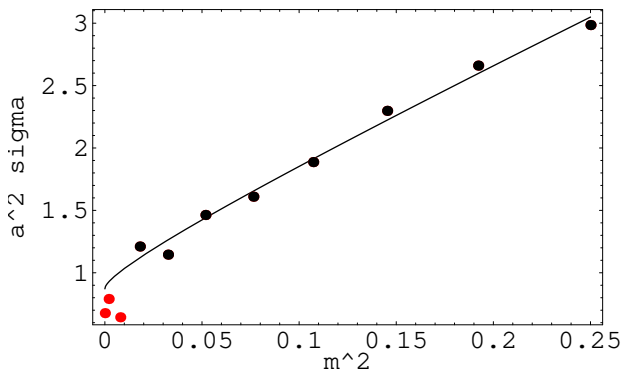
String tension in units of the coupling: sqrt(string tension)/(g^2 N). This is to be compared with the 0.1974(12) of Teper.

```
In[20]:= Map[{#[[6]] / #[[7]], (#[[16]])^(-1 / 2) (#[[19]])^(3 / 2) / #[[7]]}&,
          Select[data[[2]], (5 < #[[24]] < 25)&]]

Out[20]= {{0.018006, 0.212789}, {0.032492, 0.220207}, {0.051777, 0.196235},
          {0.076429, 0.188976}, {0.10724, 0.177135}, {0.145309, 0.162832},
          {0.192168, 0.153717}, {0.25, 0.147421}}
```

Lattice spacing a^2 in units of the string tension.

```
In[21]:= a2 = Map[{#[[6]] / #[[7]], #[[16]] / #[[19]]}&, Select[data[[2]], (5 < #[[24]] < 25)&]]

Out[21]= {{0.018006, 1.21048}, {0.032492, 1.14536}, {0.051777, 1.46318}, {0.076429, 1.60989},
          {0.10724, 1.887}, {0.145309, 2.29763}, {0.192168, 2.66022}, {0.25, 2.9851}}

In[22]:= fittinga2 = Fit[Drop[Drop[a2, 0], 0], {1, Sqrt[x], x}, x]

        General::spell1 :
         Possible spelling error: new symbol name "fittinga2" is similar to existing symbol "fitting2".

Out[22]= 0.871167 + 0.946635 √x + 6.81871 x
```

```
In[23]:= Show[Plot[fittinga2, {x, 0, 1 / 4}, DisplayFunction -> Identity],
          ListPlot[Map[{#[[6]] / #[[7]], #[[16]] / #[[19]]}&, data[[2]]],
           PlotStyle -> {PointSize[.02], Hue[0]}, DisplayFunction -> Identity],
          ListPlot[a2, PlotStyle -> {PointSize[.02]}, DisplayFunction -> Identity],
          DisplayFunction -> $DisplayFunction, Axes -> False, Frame -> True,
          FrameLabel -> {"m^2", "a^2 sigma"}]
```




```
Out[23]= - Graphics -
```

Casimir term for winding modes. This is to be compared to string theory result of $-\pi/3$ = -1.0472.

```
In[24]:= a2 = Map[{#[[6]] / #[[7]], #[[17]] / #[[19]]}&, Select[data[[2]], (5 < #[[24]] < 20)&]]

Out[24]= {{0.018006, -2.2095}, {0.032492, -2.21963}, {0.051777, -2.09462},
          {0.076429, -2.01359}, {0.145309, -1.64323}, {0.192168, -1.5948}, {0.25, -1.50526}}
```