■ Analyze data(new style March 1998)

Raw data from C program. The following gives the various parameters and truncations used in the numerical calculation.

```
In[37]:= data[[1]]

Out[37]= {******* Eigenvalues for the 2+1 transverse lattice  *******,
          Couplings:  m^2, g^2 N/a, la_1/a, la_2/a, la_3/a, tau,
                       0      1       2       3       4      5,
          Use chi^2 fit with 12 criteria, and tolerance 0.001.,
          Overall scale from fitting lowest state to lattice value.,
          2 parity doublets with fractional errors 2 0.5.,
          Spectrum for P_perp a = ( 0)  ( 0.25) ,
            using (# states, o, multiplet, c^2 error for each) =,
            (4, 1 & -1, 1 & 2, 0.5 2 0.5 0.5),   (4, 1 & -1, 2 & 1, 0.5 0.5 0.5 0.5).,
          Spectra extrapolated using (K,p) = (18/2,6)  (18/2,8) ,
            (20/2,6)  (20/2,8)  (24/2,6)  (32/2,6) .,
          Winding potential using (n,K,p) = ( 2,20/2,4)  ( 2,20/2,6) ,
            ( 2,24/2,4)  ( 2,28/2,4)  ( 3,21/2,5)  ( 3,21/2,7)  ( 3,23/2,5) ,
            ( 3,27/2,5)  ( 4,20/2,6)  ( 4,20/2,8)  ( 4,22/2,6)  ( 4,26/2,6) .,
          Heavy potential determined using (n,K,p,K_max) = ( 0,-32/2,2,4) ,
            ( 0,-32/2,4,4)  ( 0,-32/2,2,5)  ( 0,-34/2,2,4)  ( 0,-44/2,2,5) ,
            ( 0,-60/2,2,4) ,,   L = 3 4 6 (all in g^2 N/a units); with relative error 0.25.,
          Roundness determined using (n,K,p,K_max) = ( 1,-19/2,3,4) ,
            ( 1,-19/2,5,4)  ( 1,-19/2,3,5)  ( 1,-33/2,3,4)  ( 1,-33/2,3,5) ,
            ( 1,-49/2,3,4) ,,   L=3 and error 0.3 (all in g^2 N/a units).,
          p-extrapolation using (n,K,p) = ( 1,19/2,3)  ( 1,25/2,3) ,
            ( 1,39/2,3)  ( 1,19/2,5)  ( 1,25/2,5)  ( 1,23/2,7) ., Result format:,
           fit info, # steps, chi^2, p damping, and scale g^2 N/(a sigma);,
           the 6 couplings  (g^2 N/a units) and which--if any--were fit;,
           winding and longitudinal string tension fits;,
           n=1 L=3 eigenvalue and derived value (g^2 N/a units);,
           the rescaled spectrum for each P_perp*a and c^2 values.}
```

An example data point:

```
In[38]:= data[[2, -3]]

Out[38]= {2, 50, 21.0629, -1.76966, 4.27614, 0.145309, 1., -0.203064, -0.139567, 2.99797,
          -1.57487, 2, 3, 4, 5, 0.566838, -0.40549, -0.032596, 0.24906, -0.427579, 0.164112,
          0.671791, 0.705716, 16.5242, 16.5429, 0.726439, 30.979, 30.9904, 0.441706, 35.3867,
          35.3743, -0.480145, 38.1406, 38.152, 0.441531, 23.4565, 23.4621, 0.216376,
          49.3642, 49.3686, 0.16996, 53.7948, 53.8027, 0.307571, 62.4557, 62.4578, 0.081759}
```

Eigenvalue of ground state

```
In[39]:= ground = Map[(#[[{6, -24 / 2}]])&, data[[2]]]

Out[39]= {{0, 25.0985}, {0.001968, 21.4758}, {0.007919, 23.0105}, {0.018006, 23.7026},
          {0.032492, 20.3607}, {0.051777, 23.5037}, {0.076429, 22.9905}, {0.10724, 18.0205},
          {0.145309, 23.4565}, {0.192168, 21.993}, {0.25, 18.8197}}
```

lambda_1 along trajectory: lambda_1/(g^2 N) vs m^2

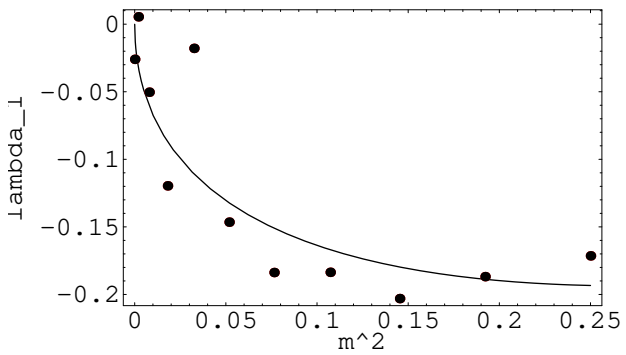
```
In[40]:= la1 = Map[(#[[{6, 8}]] / #[[7]])&, Select[data[[2]], (5 < #[[24]] < 25)&]]

Out[40]= {{0, -0.026002}, {0.001968, 0.005452}, {0.007919, -0.050322}, {0.018006, -0.119628},
          {0.032492, -0.017895}, {0.051777, -0.146483}, {0.076429, -0.183774},
          {0.10724, -0.183605}, {0.145309, -0.203064}, {0.192168, -0.186819}, {0.25, -0.171446}}

In[41]:= fitting1 = Fit[Drop[Drop[la1, 0], 0], {Sqrt[x], x}, x]

Out[41]= -0.743009 √x + 0.712565 x
```

```
In[42]:= Show[Plot[fitting1, {x, 0, 1 / 4}, DisplayFunction -> Identity],
          ListPlot[Map[(#[[{6, 8}]] / #[[7]])&, data[[2]]],
           PlotStyle -> {PointSize[.02], Hue[0]}, DisplayFunction -> Identity],
          ListPlot[la1, PlotStyle -> {PointSize[.02]}, DisplayFunction -> Identity],
          DisplayFunction -> $DisplayFunction, Axes -> False, Frame -> True,
          FrameLabel -> {"m^2", "lambda_1"}]
```




```
Out[42]= - Graphics -
```

lambda_2 along trajectory: lambda_2/(g^2 N) vs m^2

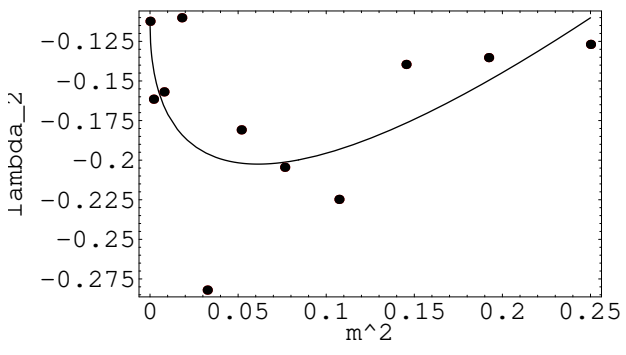
```
In[43]:= la2 = Map[(#[[{6, 9}]] / #[[7]])&, Select[data[[2]], (5 < #[[24]] < 25)&]]

Out[43]= {{0, -0.112324}, {0.001968, -0.161495}, {0.007919, -0.156888}, {0.018006, -0.110098},
          {0.032492, -0.282032}, {0.051777, -0.180874}, {0.076429, -0.204482},
          {0.10724, -0.224765}, {0.145309, -0.139567}, {0.192168, -0.135258}, {0.25, -0.126903}}

In[44]:= fitting2 = Fit[Drop[Drop[la2, 0], 0], {1, Sqrt[x], x}, x]

Out[44]= -0.113517 - 0.718566 √x + 1.45073 x

In[45]:= Show[Plot[fitting2, {x, 0, 1 / 4}, DisplayFunction -> Identity],
          ListPlot[Map[(#[[{6, 9}]] / #[[7]])&, data[[2]]],
           PlotStyle -> {PointSize[.02], Hue[0]}, DisplayFunction -> Identity],
          ListPlot[la2, PlotStyle -> {PointSize[.02]}, DisplayFunction -> Identity],
          DisplayFunction -> $DisplayFunction, Axes -> False, Frame -> True,
          FrameLabel -> {"m^2", "lambda_2"}]
```




```
Out[45]= - Graphics -
```

lambda_3 along trajectory: lambda_3/(g^2 N) vs m^2

```
In[46]:= la3 = Map[(#[[{6, 10}]] / #[[7]])&, Select[data[[2]], (5 < #[[24]] < 25)&]]

Out[46]= {{0, 451.492}, {0.001968, 625.366}, {0.007919, 1388.1}, {0.018006, 73.0956},
          {0.032492, 71.4145}, {0.051777, 3.86304}, {0.076429, 4.4978}, {0.10724, 7.61027},
          {0.145309, 2.99797}, {0.192168, 3.34441}, {0.25, 4.99416}}
```

tau along trajectory: tau/(g^2 N) vs m^2

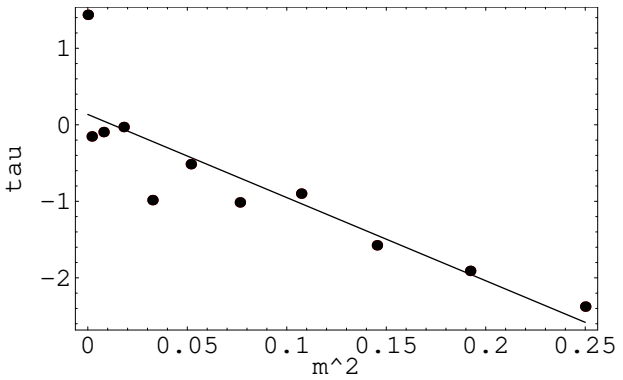
```
In[47]:= tau = Map[(#[[{6, 11}]] / #[[7]])&, Select[data[[2]], (5 < #[[24]] < 25)&]]

Out[47]= {{0, 1.43694}, {0.001968, -0.152124}, {0.007919, -0.095116}, {0.018006, -0.029268},
          {0.032492, -0.984411}, {0.051777, -0.514688}, {0.076429, -1.01412},
          {0.10724, -0.899282}, {0.145309, -1.57487}, {0.192168, -1.90906}, {0.25, -2.37566}}

In[48]:= fittingtau = Fit[Drop[Drop[tau, 0], 0], {1, x}, x]

Out[48]= 0.135156 - 10.8664 x

In[49]:= Show[Plot[fittingtau, {x, 0, 1 / 4}, DisplayFunction -> Identity],
          ListPlot[Map[(#[[{6, 11}]] / #[[7]])&, data[[2]]],
           PlotStyle -> {PointSize[.02], Hue[0]}, DisplayFunction -> Identity],
          ListPlot[tau, PlotStyle -> {PointSize[.02]}, DisplayFunction -> Identity],
          DisplayFunction -> $DisplayFunction, Axes -> False, Frame -> True,
          FrameLabel -> {"m^2", "tau"}]
```



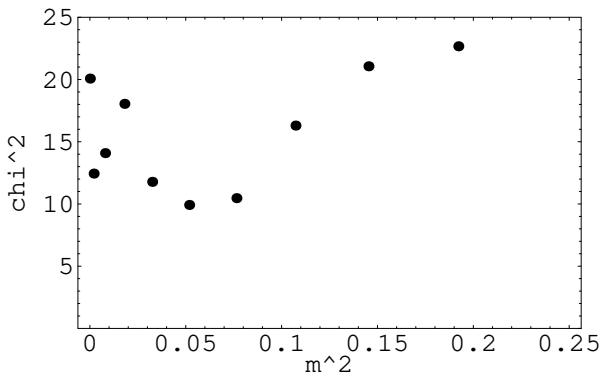

```
Out[49]= - Graphics -
```

chi^2 of fit

```
In[50]:= chi2 = Map[{#[[6]] / #[[7]], #[[3]]}&, Select[data[[2]], (5 < #[[24]] < 25)&]]

Out[50]= {{0, 20.0744}, {0.001968, 12.4411}, {0.007919, 14.0854}, {0.018006, 18.0455},
          {0.032492, 11.777}, {0.051777, 9.92116}, {0.076429, 10.4674}, {0.10724, 16.2986},
          {0.145309, 21.0629}, {0.192168, 22.6703}, {0.25, 25.5585}}
```

```
In[51]:= ListPlot[chi2, PlotRange -> {0, 25}, PlotStyle -> {PointSize[.02]}, Axes -> False,
          Frame -> True, FrameLabel -> {"m^2", "chi^2"}]
```




```
Out[51]= - Graphics -
```

Parameter for exponential damping with particle truncation: c_p vs m^2

```
In[52]:= Map[{#[[6]] / #[[7]], #[[4]]}&, Select[data[[2]], (5 < #[[24]] < 25)&]]

Out[52]= {{0, -1.46248}, {0.001968, -1.55477}, {0.007919, -1.34331}, {0.018006, -1.27155},
          {0.032492, -1.27809}, {0.051777, -1.26073}, {0.076429, -1.20063},
          {0.10724, -1.39747}, {0.145309, -1.76966}, {0.192168, -2.06959}, {0.25, -2.25707}}
```

String tension in units of the coupling: sqrt(string tension)/(g^2 N). This is to be compared with the 0.1974(12) of Teper.

```
In[53]:= Map[{#[[6]] / #[[7]], (#[[16]])^(-1 / 2) (#[[19]])^(3 / 2) / #[[7]]}&,
          Select[data[[2]], (5 < #[[24]] < 25)&]]

Out[53]= {{0, 0.262823}, {0.001968, 0.237296}, {0.007919, 0.226719}, {0.018006, 0.208879},
          {0.032492, 0.219669}, {0.051777, 0.197435}, {0.076429, 0.189047},
          {0.10724, 0.178462}, {0.145309, 0.165092}, {0.192168, 0.159859}, {0.25, 0.157684}}
```

Lattice spacing a^2 in units of the string tension.

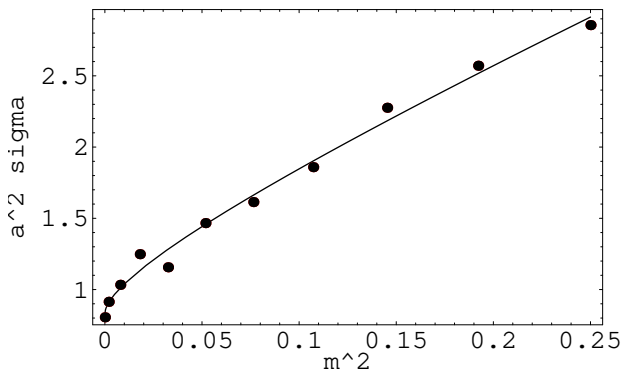
```
In[54]:= a2 = Map[{#[[6]] / #[[7]], #[[16]] / #[[19]]}&, Select[data[[2]], (5 < #[[24]] < 25)&]]

Out[54]= {{0, 0.805337}, {0.001968, 0.914033}, {0.007919, 1.03318}, {0.018006, 1.24821},
          {0.032492, 1.15627}, {0.051777, 1.46638}, {0.076429, 1.61406}, {0.10724, 1.85899},
          {0.145309, 2.27591}, {0.192168, 2.57087}, {0.25, 2.85607}}

In[55]:= fittinga2 = Fit[Drop[Drop[a2, 0], 0], {1, Sqrt[x], x}, x]

Out[55]= 0.827982 + 1.5939 √x + 5.15021 x
```

```
In[56]:= Show[Plot[fittinga2, {x, 0, 1 / 4}, DisplayFunction -> Identity],
          ListPlot[Map[{#[[6]] / #[[7]], #[[16]] / #[[19]]}&, data[[2]]],
           PlotStyle -> {PointSize[.02], Hue[0]}, DisplayFunction -> Identity],
          ListPlot[a2, PlotStyle -> {PointSize[.02]}, DisplayFunction -> Identity],
          DisplayFunction -> $DisplayFunction, Axes -> False, Frame -> True,
          FrameLabel -> {"m^2", "a^2 sigma"}]
```




```
Out[56]= - Graphics -
```

Casimir term for winding modes. This is to be compared to string theory result of $-\pi/3$ = -1.0472.

```
In[57]:= a2 = Map[{#[[6]] / #[[7]], #[[17]] / #[[19]]}&, Select[data[[2]], (5 < #[[24]] < 20)&]]

Out[57]= {{0, -1.8933}, {0.001968, -2.0395}, {0.007919, -2.09993}, {0.018006, -2.16834},
          {0.032492, -2.14329}, {0.051777, -2.0384}, {0.076429, -1.97764},
          {0.10724, -1.77614}, {0.145309, -1.62808}, {0.192168, -1.52886}, {0.25, -1.44143}}
```