■ Find chiral 4–fermioninteraction to leading order, with flavor

```
In[172]:= u[x_, f_] :=
            1 / Sqrt[2 Pi] (b[kf, f] Exp[-I kf x] + sigma[1] ** Conjugate[d[kf, f]] Exp[I kf x]);
          kappaa /: Conjugate[kappaa[a_]] := kappaa[a]
          kappas /: Conjugate[kappas[a_]] := kappas[a]
          muf /: Conjugate[muf[a_]] := muf[a]
```

Xi is used for the Yukawa interactions and the 4–fermioninteractions

```
In[176]:= xi[x_, f_] =
            Expand[muf[f] / Sqrt[2] sigma[1] ** u[x, f] + kappaa[f] / Sqrt[2] sigma[1] ** zeta[rr] **
                   m[x, rr] ** u[x, f] - kappas[f] / Sqrt[2] sigma[1] ** m[x, sumrr] ** u[x, f] //.
                prodexpand /. {zeta[rho_] ** a[c_, -rho_] → -zeta[rho] ** a[c, rho],
                a[b_, -sumrr] → a[b, sumrr]} //. prodexpand];
```

We need the definition of xi from the Yukawa interactions

```
In[177]:= cxi[x_, f_] =
           Conjugate[xi[x, f]] /. {Conjugate[x] → x, Conjugate[ka] → ka, Conjugate[kf] → kf,
                Conjugate[ka^n_] → ka^n} /.
              {ka → kka, kf → kks, rr → ll, sumrr → sumll} //. prodexpand //. sigmazeta
```

```
Out[177]= e^(-I kks x) d[kks, f] muf[f] / (2 Sqrt[π]) + e^(I kks x) muf[f] Conjugate[b[kks, f]] ** sigma[1] /
            (2 Sqrt[π]) -
          e^(-I kka x - I kks x) kappas[f] d[kks, f] ** a[kka, sumll] /
            (4 Sqrt[kka] π) -
          e^(I kka x - I kks x) kappas[f] d[kks, f] ** Conjugate[a[kka, sumll]] /
            (4 Sqrt[kka] π) -
          e^(-I kka x + I kks x) kappas[f] Conjugate[b[kks, f]] ** sigma[1] ** a[kka, sumll] /
            (4 Sqrt[kka] π) -
          e^(I kka x + I kks x) kappas[f] Conjugate[b[kks, f]] ** sigma[1] ** Conjugate[a[kka, sumll]] /
            (4 Sqrt[kka] π) +
          e^(-I kka x + I kks x) kappaa[f] Conjugate[b[kks, f]] ** zeta[ll] ** sigma[1] ** a[kka, ll] /
            (4 Sqrt[kka] π) -
          1 / (4 Sqrt[kka] π) (e^(I kka x + I kks x) kappaa[f]
            Conjugate[b[kks, f]] ** zeta[ll] ** sigma[1] ** Conjugate[a[kka, ll]]) +
          e^(-I kka x - I kks x) kappaa[f] d[kks, f] ** sigma[1] ** zeta[ll] ** sigma[1] ** a[kka, ll] /
            (4 Sqrt[kka] π) -
          1 / (4 Sqrt[kka] π) (e^(I kka x - I kks x) kappaa[f]
            d[kks, f] ** sigma[1] ** zeta[ll] ** sigma[1] ** Conjugate[a[kka, ll]])
```

```
In[178]:= conju[x_, f_] = Conjugate[u[x, f]] /. {Conjugate[x] → x, Conjugate[kf] → kf} /. kf → kks
```

```
Out[178]= (e^(I kks x) Conjugate[b[kks, f]] + e^(-I kks x) d[kks, f] ** sigma[1]) / Sqrt[2 π]
```

The chiral four–fermioninteraction in the Hamiltonian is gamma qq**qq/2 where qq is:

```
In[179]:= qq[x_, f1_, f2_] =
           relabel[Expand[I conju[x, f1] ** sigma[2] ** Integrate[xi[x, f2], x] - I
                  Integrate[cxi[x, f1], x] ** sigma[2] ** u[x, f2] //.
                prodexpand //. sigmazeta]] //. prodfactor
```

Out[179]=

$$\frac{1}{2\sqrt{2}\ \text{kl1}\ \text{kl2}\ \pi}\Big(e^{i\,(\text{kl1}+\text{kl2})\,x}\ (\text{kl2}\ \text{muf}[\text{f1}]+\text{kl1}\ \text{muf}[\text{f2}])$$
$$\text{Conjugate}[\text{b}[\text{kl1},\text{f1}]]\ **\ \text{sigma}[2]\ **\ \text{Conjugate}[\text{d}[\text{kl2},\text{f2}]]\Big)+$$
$$\frac{i\,e^{i\,(\text{kl1}-\text{kr1})\,x}\ (-\text{kr1}\ \text{muf}[\text{f1}]+\text{kl1}\ \text{muf}[\text{f2}])\ \text{Conjugate}[\text{b}[\text{kl1},\text{f1}]]\ **\ \text{sigma}[3]\ **\ \text{b}[\text{kr1},\text{f2}]}{2\sqrt{2}\ \text{kl1}\ \text{kr1}\ \pi}$$
$$-\frac{i\,e^{i\,(\text{kl1}-\text{kr1})\,x}\ (\text{kl1}\ \text{muf}[\text{f1}]-\text{kr1}\ \text{muf}[\text{f2}])\ \text{d}[\text{kr1},\text{f1}]\ **\ \text{sigma}[3]\ **\ \text{Conjugate}[\text{d}[\text{kl1},\text{f2}]]}{2\sqrt{2}\ \text{kl1}\ \text{kr1}\ \pi}$$
$$+\frac{e^{-i\,(\text{kr1}+\text{kr2})\,x}\ (\text{kr1}\ \text{muf}[\text{f1}]+\text{kr2}\ \text{muf}[\text{f2}])\ \text{d}[\text{kr2},\text{f1}]\ **\ \text{sigma}[2]\ **\ \text{b}[\text{kr1},\text{f2}]}{2\sqrt{2}\ \text{kr1}\ \text{kr2}\ \pi}+$$
$$\Big(e^{i\,(\text{kl1}+\text{kl2}-\text{kr1})\,x}\ ((-\text{kl2}+\text{kr1})\ \text{kappas}[\text{f1}]+(-\text{kl1}+\text{kr1})\ \text{kappas}[\text{f2}])$$
$$\text{Conjugate}[\text{b}[\text{kl1},\text{f1}]]\ **\ \text{sigma}[2]\ **\ \text{a}[\text{kr1},\text{s1}]\ **\ \text{Conjugate}[\text{d}[\text{kl2},\text{f2}]]\Big)\Big/$$
$$\Big(4\sqrt{2}\ \sqrt{\text{kr1}}\ (-\text{kl1}+\text{kr1})\ (-\text{kl2}+\text{kr1})\ \pi^{3/2}\Big)-$$
$$\Big(e^{i\,(\text{kl1}+\text{kl2}+\text{kl3})\,x}\ ((\text{kl2}+\text{kl3})\ \text{kappas}[\text{f1}]+(\text{kl1}+\text{kl2})\ \text{kappas}[\text{f2}])\ \text{Conjugate}[\text{b}[\text{kl1},\text{f1}]]\ **$$
$$\text{sigma}[2]\ **\ \text{Conjugate}[\text{a}[\text{kl2},\text{s1}]]\ **\ \text{Conjugate}[\text{d}[\text{kl3},\text{f2}]]\Big)\Big/$$
$$\Big(4\sqrt{2}\ \sqrt{\text{kl2}}\ (\text{kl1}+\text{kl2})\ (\text{kl2}+\text{kl3})\ \pi^{3/2}\Big)-$$
$$\Big(i\,e^{i\,(\text{kl1}-\text{kr1}-\text{kr2})\,x}\ ((\text{kr1}+\text{kr2})\ \text{kappas}[\text{f1}]+(-\text{kl1}+\text{kr2})\ \text{kappas}[\text{f2}])$$
$$\text{Conjugate}[\text{b}[\text{kl1},\text{f1}]]\ **\ \text{sigma}[3]\ **\ \text{a}[\text{kr2},\text{s1}]\ **\ \text{b}[\text{kr1},\text{f2}]\Big)\Big/$$
$$\Big(4\sqrt{2}\ \sqrt{\text{kr2}}\ (-\text{kl1}+\text{kr2})\ (\text{kr1}+\text{kr2})\ \pi^{3/2}\Big)+$$
$$\Big(i\,e^{i\,(\text{kl1}+\text{kl2}-\text{kr1})\,x}\ ((\text{kl2}-\text{kr1})\ \text{kappas}[\text{f1}]+(\text{kl1}+\text{kl2})\ \text{kappas}[\text{f2}])$$
$$\text{Conjugate}[\text{b}[\text{kl1},\text{f1}]]\ **\ \text{sigma}[3]\ **\ \text{Conjugate}[\text{a}[\text{kl2},\text{s1}]]\ **\ \text{b}[\text{kr1},\text{f2}]\Big)\Big/$$
$$\Big(4\sqrt{2}\ \sqrt{\text{kl2}}\ (\text{kl1}+\text{kl2})\ (\text{kl2}-\text{kr1})\ \pi^{3/2}\Big)-$$
$$\Big(i\,e^{i\,(\text{kl1}+\text{kl2}-\text{kr1})\,x}\ ((\text{kl1}+\text{kl2})\ \text{kappas}[\text{f1}]+(\text{kl1}-\text{kr1})\ \text{kappas}[\text{f2}])$$
$$\text{d}[\text{kr1},\text{f1}]\ **\ \text{sigma}[3]\ **\ \text{Conjugate}[\text{a}[\text{kl1},\text{s1}]]\ **\ \text{Conjugate}[\text{d}[\text{kl2},\text{f2}]]\Big)\Big/$$
$$\Big(4\sqrt{2}\ \sqrt{\text{kl1}}\ (\text{kl1}+\text{kl2})\ (\text{kl1}-\text{kr1})\ \pi^{3/2}\Big)+$$
$$\Big(e^{-i\,(-\text{kl1}+\text{kr1}+\text{kr2})\,x}\ ((\text{kl1}-\text{kr1})\ \text{kappas}[\text{f1}]+(\text{kl1}-\text{kr2})\ \text{kappas}[\text{f2}])$$
$$\text{d}[\text{kr2},\text{f1}]\ **\ \text{sigma}[2]\ **\ \text{Conjugate}[\text{a}[\text{kl1},\text{s1}]]\ **\ \text{b}[\text{kr1},\text{f2}]\Big)\Big/$$
$$\Big(4\sqrt{2}\ \sqrt{\text{kl1}}\ (\text{kl1}-\text{kr1})\ (\text{kl1}-\text{kr2})\ \pi^{3/2}\Big)+$$
$$\Big(i\,e^{i\,(\text{kl1}-\text{kr1}-\text{kr2})\,x}\ ((-\text{kl1}+\text{kr1})\ \text{kappas}[\text{f1}]+(\text{kr1}+\text{kr2})\ \text{kappas}[\text{f2}])$$
$$\text{d}[\text{kr2},\text{f1}]\ **\ \text{sigma}[3]\ **\ \text{a}[\text{kr1},\text{s1}]\ **\ \text{Conjugate}[\text{d}[\text{kl1},\text{f2}]]\Big)\Big/$$
$$\Big(4\sqrt{2}\ \sqrt{\text{kr1}}\ (-\text{kl1}+\text{kr1})\ (\text{kr1}+\text{kr2})\ \pi^{3/2}\Big)-$$
$$\Big(e^{-i\,(\text{kr1}+\text{kr2}+\text{kr3})\,x}\ ((\text{kr1}+\text{kr2})\ \text{kappas}[\text{f1}]+(\text{kr2}+\text{kr3})\ \text{kappas}[\text{f2}])$$
$$\text{d}[\text{kr3},\text{f1}]\ **\ \text{sigma}[2]\ **\ \text{a}[\text{kr2},\text{s1}]\ **\ \text{b}[\text{kr1},\text{f2}]\Big)\Big/$$
$$\Big(4\sqrt{2}\ \sqrt{\text{kr2}}\ (\text{kr1}+\text{kr2})\ (\text{kr2}+\text{kr3})\ \pi^{3/2}\Big)+$$
$$\Big(e^{i\,(\text{kl1}+\text{kl2}-\text{kr1})\,x}\ ((\text{kl2}-\text{kr1})\ \text{kappaa}[\text{f1}]+(\text{kl1}-\text{kr1})\ \text{kappaa}[\text{f2}])\ \text{Conjugate}[\text{b}[\text{kl1},\text{f1}]]\ **$$
$$\text{zeta}[\text{s1}]\ **\ \text{sigma}[2]\ **\ \text{a}[\text{kr1},\text{s1}]\ **\ \text{Conjugate}[\text{d}[\text{kl2},\text{f2}]]\Big)\Big/$$
$$\Big(4\sqrt{2}\ \sqrt{\text{kr1}}\ (-\text{kl1}+\text{kr1})\ (-\text{kl2}+\text{kr1})\ \pi^{3/2}\Big)-$$
$$\Big(e^{i\,(\text{kl1}+\text{kl2}+\text{kl3})\,x}\ ((\text{kl2}+\text{kl3})\ \text{kappaa}[\text{f1}]+(\text{kl1}+\text{kl2})\ \text{kappaa}[\text{f2}])\ \text{Conjugate}[\text{b}[\text{kl1},\text{f1}]]\ **$$
$$\text{zeta}[\text{s1}]\ **\ \text{sigma}[2]\ **\ \text{Conjugate}[\text{a}[\text{kl2},\text{s1}]]\ **\ \text{Conjugate}[\text{d}[\text{kl3},\text{f2}]]\Big)\Big/$$
$$\Big(4\sqrt{2}\ \sqrt{\text{kl2}}\ (\text{kl1}+\text{kl2})\ (\text{kl2}+\text{kl3})\ \pi^{3/2}\Big)+$$
$$\Big(i\,e^{i\,(\text{kl1}-\text{kr1}-\text{kr2})\,x}\ ((\text{kr1}+\text{kr2})\ \text{kappaa}[\text{f1}]+(-\text{kl1}+\text{kr2})\ \text{kappaa}[\text{f2}])$$
$$\text{Conjugate}[\text{b}[\text{kl1},\text{f1}]]\ **\ \text{zeta}[\text{s1}]\ **\ \text{sigma}[3]\ **\ \text{a}[\text{kr2},\text{s1}]\ **\ \text{b}[\text{kr1},\text{f2}]\Big)\Big/$$
$$\Big(4\sqrt{2}\ \sqrt{\text{kr2}}\ (-\text{kl1}+\text{kr2})\ (\text{kr1}+\text{kr2})\ \pi^{3/2}\Big)+$$
$$\Big(i\,e^{i\,(\text{kl1}+\text{kl2}-\text{kr1})\,x}\ ((\text{kl2}-\text{kr1})\ \text{kappaa}[\text{f1}]+(\text{kl1}+\text{kl2})\ \text{kappaa}[\text{f2}])$$
$$\text{Conjugate}[\text{b}[\text{kl1},\text{f1}]]\ **\ \text{zeta}[\text{s1}]\ **\ \text{sigma}[3]\ **\ \text{Conjugate}[\text{a}[\text{kl2},\text{s1}]]\ **$$
$$\text{b}[\text{kr1},\text{f2}]\Big)\Big/\Big(4\sqrt{2}\ \sqrt{\text{kl2}}\ (\text{kl1}+\text{kl2})\ (\text{kl2}-\text{kr1})\ \pi^{3/2}\Big)+$$
$$\Big(e^{-i\,(-\text{kl1}+\text{kr1}+\text{kr2})\,x}\ ((-\text{kl1}+\text{kr1})\ \text{kappaa}[\text{f1}]+(-\text{kl1}+\text{kr2})\ \text{kappaa}[\text{f2}])$$
$$\text{d}[\text{kr2},\text{f1}]\ **\ \text{sigma}[2]\ **\ \text{zeta}[\text{s1}]\ **\ \text{Conjugate}[\text{a}[\text{kl1},\text{s1}]]\ **\ \text{b}[\text{kr1},\text{f2}]\Big)\Big/$$

```
   (4 Sqrt[2] Sqrt[kl1] (kl1 - kr1) (kl1 - kr2) π^(3/2)) -
  (e^(-i (kr1+kr2+kr3) x) ((kr1 + kr2) kappaa[f1] + (kr2 + kr3) kappaa[f2])
     d[kr3, f1] ** sigma[2] ** zeta[s1] ** a[kr2, s1] ** b[kr1, f2]) /
   (4 Sqrt[2] Sqrt[kr2] (kr1 + kr2) (kr2 + kr3) π^(3/2)) -
  (e^(i (kl1+kl2-kr1) x) ((kl1 + kl2) kappaa[f1] + (kl1 - kr1) kappaa[f2]) d[kr1, f1] ** sigma[1] **
     zeta[s1] ** sigma[2] ** Conjugate[a[kl1, s1]] ** Conjugate[d[kl2, f2]]) /
   (4 Sqrt[2] Sqrt[kl1] (kl1 + kl2) (kl1 - kr1) π^(3/2)) -
  (e^(i (kl1-kr1-kr2) x) ((-kl1 + kr1) kappaa[f1] + (kr1 + kr2) kappaa[f2]) d[kr2, f1] **
     sigma[1] ** zeta[s1] ** sigma[2] ** a[kr1, s1] ** Conjugate[d[kl1, f2]]) /
   (4 Sqrt[2] Sqrt[kr1] (-kl1 + kr1) (kr1 + kr2) π^(3/2))
```

■ Select terms that are leading order in N for mesons ("pinching" terms):

```
In[180]:= Clear[qqpinch]; qqpinch[x_, f1_, f2_] = qq[x, f1, f2] /.
            a_NonCommutativeMultiply :> Block[{non = a //. {sigma[_] ** c_ → c, zeta[_] ** c_ → c}},
              0 /; Count[non, Conjugate[_]] != Length[non] && Count[non, Conjugate[_]] != 0]
```

```
Out[180]= 1/(2 Sqrt[2] kl1 kl2 π) (e^(i (kl1+kl2) x) (kl2 muf[f1] + kl1 muf[f2])
     Conjugate[b[kl1, f1]] ** sigma[2] ** Conjugate[d[kl2, f2]]) +
   (e^(-i (kr1+kr2) x) (kr1 muf[f1] + kr2 muf[f2]) d[kr2, f1] ** sigma[2] ** b[kr1, f2]) /
     (2 Sqrt[2] kr1 kr2 π) -
   (e^(i (kl1+kl2+kl3) x) ((kl2 + kl3) kappas[f1] + (kl1 + kl2) kappas[f2]) Conjugate[b[kl1, f1]] **
     sigma[2] ** Conjugate[a[kl2, s1]] ** Conjugate[d[kl3, f2]]) /
   (4 Sqrt[2] Sqrt[kl2] (kl1 + kl2) (kl2 + kl3) π^(3/2)) -
  (e^(-i (kr1+kr2+kr3) x) ((kr1 + kr2) kappas[f1] + (kr2 + kr3) kappas[f2])
     d[kr3, f1] ** sigma[2] ** a[kr2, s1] ** b[kr1, f2]) /
   (4 Sqrt[2] Sqrt[kr2] (kr1 + kr2) (kr2 + kr3) π^(3/2)) -
  (e^(i (kl1+kl2+kl3) x) ((kl2 + kl3) kappaa[f1] + (kl1 + kl2) kappaa[f2]) Conjugate[b[kl1, f1]] **
     zeta[s1] ** sigma[2] ** Conjugate[a[kl2, s1]] ** Conjugate[d[kl3, f2]]) /
   (4 Sqrt[2] Sqrt[kl2] (kl1 + kl2) (kl2 + kl3) π^(3/2)) -
  (e^(-i (kr1+kr2+kr3) x) ((kr1 + kr2) kappaa[f1] + (kr2 + kr3) kappaa[f2])
     d[kr3, f1] ** sigma[2] ** zeta[s1] ** a[kr2, s1] ** b[kr1, f2]) /
   (4 Sqrt[2] Sqrt[kr2] (kr1 + kr2) (kr2 + kr3) π^(3/2))
```

```
In[181]:= (*put in the coupling constant later*)
          expr1pinch = relabel[Map[Distribute, qqpinch[x, f1, f2] /. {kl1 → kkl1,
                kl2 → kkl2, kl3 → kkl3, kr1 → kkr1, kr2 → kkr2, kr3 → kkr3, s1 → ss1}] **
              Map[Distribute, qqpinch[x, f2, f1]] //. prodexpand] //. prodfactor;
```

expr2 contians all of the "pinching" interactions in the Hamiltonian.

```
In[182]:= (*gamma is the coupling constant*)
          expr2pinch =
           gamma / 2 * simpint[int[expr1pinch, {x}] //. intsumexpand /. Exp[x_] :> Exp[Factor[x]] /.
               int[Exp[a_ b_], {b_}] → 2 Pi delta[I a] /. delta[_?neverzero] → 0, kl1]
```

```
Out[182]= 1/2 gamma (1/(4 kl1 kl2 kr1 kr2 π)
     (delta[-kl1 - kl2 + kr1 + kr2] (kl2 muf[f1] + kl1 muf[f2]) (kr2 muf[f1] + kr1 muf[f2])
       Conjugate[b[kl1, f1]] ** sigma[2] ** Conjugate[d[kl2, f2]] **
        d[kr2, f2] ** sigma[2] ** b[kr1, f1]) + 1/(4 kl1 kl2 kr1 kr2 π)
     (delta[-kl1 - kl2 + kr1 + kr2] (kl1 muf[f1] + kl2 muf[f2])
       (kr1 muf[f1] + kr2 muf[f2]) d[kr2, f1] ** sigma[2] ** b[kr1, f2] **
```

```
   Conjugate[b[kl1, f2]] ** sigma[2] ** Conjugate[d[kl2, f1]]) -
(delta[-kl1 - kl2 - kl3 + kr1 + kr2] ((kl2 + kl3) kappas[f1] + (kl1 + kl2) kappas[f2])
   (kr2 muf[f1] + kr1 muf[f2]) Conjugate[b[kl1, f1]] ** sigma[2] ** Conjugate[
     a[kl2, s1]] ** Conjugate[d[kl3, f2]] ** d[kr2, f2] ** sigma[2] ** b[kr1, f1]) /
 (8 Sqrt[kl2] (kl1 + kl2) (kl2 + kl3) kr1 kr2 π^(3/2)) -
(delta[-kl1 - kl2 + kr1 + kr2 + kr3] ((kr2 + kr3) kappas[f1] + (kr1 + kr2) kappas[f2])
   (kl2 muf[f1] + kl1 muf[f2]) Conjugate[b[kl1, f1]] ** sigma[2] **
    Conjugate[d[kl2, f2]] ** d[kr3, f2] ** sigma[2] ** a[kr2, s1] ** b[kr1, f1]) /
 (8 kl1 kl2 Sqrt[kr2] (kr1 + kr2) (kr2 + kr3) π^(3/2)) -
(delta[-kl1 - kl2 - kl3 + kr1 + kr2] ((kl1 + kl2) kappas[f1] + (kl2 + kl3) kappas[f2])
   (kr1 muf[f1] + kr2 muf[f2]) d[kr2, f1] ** sigma[2] ** b[kr1, f2] ** Conjugate[
     b[kl1, f2]] ** sigma[2] ** Conjugate[a[kl2, s1]] ** Conjugate[d[kl3, f1]]) /
 (8 Sqrt[kl2] (kl1 + kl2) (kl2 + kl3) kr1 kr2 π^(3/2)) -
(delta[-kl1 - kl2 + kr1 + kr2 + kr3] ((kr1 + kr2) kappas[f1] + (kr2 + kr3) kappas[f2])
   (kl1 muf[f1] + kl2 muf[f2]) d[kr3, f1] ** sigma[2] ** a[kr2, s1] **
    b[kr1, f2] ** Conjugate[b[kl1, f2]] ** sigma[2] ** Conjugate[d[kl2, f1]]) /
 (8 kl1 kl2 Sqrt[kr2] (kr1 + kr2) (kr2 + kr3) π^(3/2)) +
(delta[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3] ((kl2 + kl3) kappas[f1] +
     (kl1 + kl2) kappas[f2]) ((kr2 + kr3) kappas[f1] + (kr1 + kr2) kappas[f2])
   Conjugate[b[kl1, f1]] ** sigma[2] ** Conjugate[a[kl2, s2]] **
    Conjugate[d[kl3, f2]] ** d[kr3, f2] ** sigma[2] ** a[kr2, s1] ** b[kr1, f1]) /
 (16 Sqrt[kl2] (kl1 + kl2) (kl2 + kl3) Sqrt[kr2] (kr1 + kr2) (kr2 + kr3) π^2) -
(delta[-kl1 - kl2 + kr1 + kr2 + kr3] ((kr2 + kr3) kappaa[f1] + (kr1 + kr2) kappaa[f2])
   (kl2 muf[f1] + kl1 muf[f2]) Conjugate[b[kl1, f1]] ** sigma[2] ** Conjugate[
     d[kl2, f2]] ** d[kr3, f2] ** sigma[2] ** zeta[s1] ** a[kr2, s1] ** b[kr1, f1]) /
 (8 kl1 kl2 Sqrt[kr2] (kr1 + kr2) (kr2 + kr3) π^(3/2)) - (delta[-kl1 - kl2 - kl3 + kr1 + kr2]
   ((kl2 + kl3) kappaa[f1] + (kl1 + kl2) kappaa[f2]) (kr2 muf[f1] + kr1 muf[f2])
   Conjugate[b[kl1, f1]] ** zeta[s1] ** sigma[2] ** Conjugate[a[kl2, s1]] **
    Conjugate[d[kl3, f2]] ** d[kr2, f2] ** sigma[2] ** b[kr1, f1]) /
 (8 Sqrt[kl2] (kl1 + kl2) (kl2 + kl3) kr1 kr2 π^(3/2)) - (delta[-kl1 - kl2 - kl3 + kr1 + kr2]
   ((kl1 + kl2) kappaa[f1] + (kl2 + kl3) kappaa[f2]) (kr1 muf[f1] + kr2 muf[f2])
   d[kr2, f1] ** sigma[2] ** b[kr1, f2] ** Conjugate[b[kl1, f2]] **
    zeta[s1] ** sigma[2] ** Conjugate[a[kl2, s1]] ** Conjugate[d[kl3, f1]]) /
 (8 Sqrt[kl2] (kl1 + kl2) (kl2 + kl3) kr1 kr2 π^(3/2)) +
(delta[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3] ((kl1 + kl2) kappas[f1] +
     (kl2 + kl3) kappas[f2]) ((kr1 + kr2) kappas[f1] + (kr2 + kr3) kappas[f2])
   d[kr3, f1] ** sigma[2] ** a[kr2, s2] ** b[kr1, f2] ** Conjugate[b[kl1, f2]] **
    sigma[2] ** Conjugate[a[kl2, s1]] ** Conjugate[d[kl3, f1]]) /
 (16 Sqrt[kl2] (kl1 + kl2) (kl2 + kl3) Sqrt[kr2] (kr1 + kr2) (kr2 + kr3) π^2) -
(delta[-kl1 - kl2 + kr1 + kr2 + kr3] ((kr1 + kr2) kappaa[f1] + (kr2 + kr3) kappaa[f2])
   (kl1 muf[f1] + kl2 muf[f2]) d[kr3, f1] ** sigma[2] ** zeta[s1] ** a[kr2, s1] **
    b[kr1, f2] ** Conjugate[b[kl1, f2]] ** sigma[2] ** Conjugate[d[kl2, f1]]) /
 (8 kl1 kl2 Sqrt[kr2] (kr1 + kr2) (kr2 + kr3) π^(3/2)) + (delta[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3]
   ((kr2 + kr3) kappaa[f1] + (kr1 + kr2) kappaa[f2])
   ((kl2 + kl3) kappas[f1] + (kl1 + kl2) kappas[f2]) Conjugate[b[kl1, f1]] **
    sigma[2] ** Conjugate[a[kl2, s2]] ** Conjugate[d[kl3, f2]] **
    d[kr3, f2] ** sigma[2] ** zeta[s1] ** a[kr2, s1] ** b[kr1, f1]) /
 (16 Sqrt[kl2] (kl1 + kl2) (kl2 + kl3) Sqrt[kr2] (kr1 + kr2) (kr2 + kr3) π^2) +
(delta[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3] ((kl2 + kl3) kappaa[f1] +
     (kl1 + kl2) kappaa[f2]) ((kr2 + kr3) kappas[f1] + (kr1 + kr2) kappas[f2])
   Conjugate[b[kl1, f1]] ** zeta[s2] ** sigma[2] ** Conjugate[a[kl2, s2]] **
    Conjugate[d[kl3, f2]] ** d[kr3, f2] ** sigma[2] ** a[kr2, s1] ** b[kr1, f1]) /
 (16 Sqrt[kl2] (kl1 + kl2) (kl2 + kl3) Sqrt[kr2] (kr1 + kr2) (kr2 + kr3) π^2) +
(delta[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3] ((kl1 + kl2) kappaa[f1] +
     (kl2 + kl3) kappaa[f2]) ((kr1 + kr2) kappas[f1] + (kr2 + kr3) kappas[f2])
   d[kr3, f1] ** sigma[2] ** a[kr2, s2] ** b[kr1, f2] ** Conjugate[b[kl1, f2]] **
```

```
      zeta[s1] ** sigma[2] ** Conjugate[a[kl2, s1]] ** Conjugate[d[kl3, f1]]]) /
   (16 Sqrt[kl2] (kl1 + kl2) (kl2 + kl3) Sqrt[kr2] (kr1 + kr2) (kr2 + kr3) π^2) +
  (delta[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3]
     ((kr1 + kr2) kappaa[f1] + (kr2 + kr3) kappaa[f2])
     ((kl1 + kl2) kappas[f1] + (kl2 + kl3) kappas[f2]) d[kr3, f1] ** sigma[2] **
      zeta[s2] ** a[kr2, s2] ** b[kr1, f2] ** Conjugate[b[kl1, f2]] **
      sigma[2] ** Conjugate[a[kl2, s1]] ** Conjugate[d[kl3, f1]]) /
   (16 Sqrt[kl2] (kl1 + kl2) (kl2 + kl3) Sqrt[kr2] (kr1 + kr2) (kr2 + kr3) π^2) +
  (delta[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3]
     ((kl2 + kl3) kappaa[f1] + (kl1 + kl2) kappaa[f2])
     ((kr2 + kr3) kappaa[f1] + (kr1 + kr2) kappaa[f2]) Conjugate[b[kl1, f1]] **
      zeta[s2] ** sigma[2] ** Conjugate[a[kl2, s2]] ** Conjugate[d[kl3, f2]] **
      d[kr3, f2] ** sigma[2] ** zeta[s1] ** a[kr2, s1] ** b[kr1, f1]) /
   (16 Sqrt[kl2] (kl1 + kl2) (kl2 + kl3) Sqrt[kr2] (kr1 + kr2) (kr2 + kr3) π^2) +
  (delta[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3]
     ((kl1 + kl2) kappaa[f1] + (kl2 + kl3) kappaa[f2])
     ((kr1 + kr2) kappaa[f1] + (kr2 + kr3) kappaa[f2]) d[kr3, f1] ** sigma[2] **
      zeta[s2] ** a[kr2, s2] ** b[kr1, f2] ** Conjugate[b[kl1, f2]] **
      zeta[s1] ** sigma[2] ** Conjugate[a[kl2, s1]] ** Conjugate[d[kl3, f1]]) /
   (16 Sqrt[kl2] (kl1 + kl2) (kl2 + kl3) Sqrt[kr2] (kr1 + kr2) (kr2 + kr3) π^2))
```

Just look at the term that acts in the two particle sector.

```
In[183]:= expr2pinch /. {kappas[_] → 0, kappaa[_] → 0}
```

```
Out[183]= 1/2 gamma (1/(4 kl1 kl2 kr1 kr2 π)
    (delta[-kl1 - kl2 + kr1 + kr2] (kl2 muf[f1] + kl1 muf[f2]) (kr2 muf[f1] + kr1 muf[f2])
      Conjugate[b[kl1, f1]] ** sigma[2] ** Conjugate[d[kl2, f2]] **
       d[kr2, f2] ** sigma[2] ** b[kr1, f1]) + 1/(4 kl1 kl2 kr1 kr2 π)
    (delta[-kl1 - kl2 + kr1 + kr2] (kl1 muf[f1] + kl2 muf[f2])
      (kr1 muf[f1] + kr2 muf[f2]) d[kr2, f1] ** sigma[2] ** b[kr1, f2] **
       Conjugate[b[kl1, f2]] ** sigma[2] ** Conjugate[d[kl2, f1]]))
```

■ Select terms that act on a baryon (no anti−quarks)

```
In[184]:= Clear[qqbaryon];
          qqbaryon[x_, f1_, f2_] = qq[x, f1, f2] /. d[_] → 0 /. {0 ** a_ → 0, a_ ** 0 → 0}
```

```
Out[185]= 1/(2 Sqrt[2] kl1 kl2 π) (E^(i (kl1+kl2) x) (kl2 muf[f1] + kl1 muf[f2])
     Conjugate[b[kl1, f1]] ** sigma[2] ** Conjugate[d[kl2, f2]]) +
   (i E^(i (kl1-kr1) x) (-kr1 muf[f1] + kl1 muf[f2]) Conjugate[b[kl1, f1]] ** sigma[3] ** b[kr1, f2]) /
    (2 Sqrt[2] kl1 kr1 π)
  -
   (i E^(i (kl1-kr1) x) (kl1 muf[f1] - kr1 muf[f2]) d[kr1, f1] ** sigma[3] ** Conjugate[d[kl1, f2]]) /
    (2 Sqrt[2] kl1 kr1 π)
  + (E^(-i (kr1+kr2) x) (kr1 muf[f1] + kr2 muf[f2]) d[kr2, f1] ** sigma[2] ** b[kr1, f2]) /
    (2 Sqrt[2] kr1 kr2 π) +
  (E^(i (kl1+kl2-kr1) x) ((-kl2 + kr1) kappas[f1] + (-kl1 + kr1) kappas[f2])
     Conjugate[b[kl1, f1]] ** sigma[2] ** a[kr1, s1] ** Conjugate[d[kl2, f2]]) /
   (4 Sqrt[2] Sqrt[kr1] (-kl1 + kr1) (-kl2 + kr1) π^(3/2)) -
  (E^(i (kl1+kl2+kl3) x) ((kl2 + kl3) kappas[f1] + (kl1 + kl2) kappas[f2]) Conjugate[b[kl1, f1]] **
     sigma[2] ** Conjugate[a[kl2, s1]] ** Conjugate[d[kl3, f2]]) /
```

```
   (4 Sqrt[2] Sqrt[kl2] (kl1 + kl2) (kl2 + kl3) π^(3/2)) -
  (i e^(i (kl1-kr1-kr2) x) ((kr1 + kr2) kappas[f1] + (-kl1 + kr2) kappas[f2])
     Conjugate[b[kl1, f1]] ** sigma[3] ** a[kr2, s1] ** b[kr1, f2]) /
   (4 Sqrt[2] Sqrt[kr2] (-kl1 + kr2) (kr1 + kr2) π^(3/2)) +
  (i e^(i (kl1+kl2-kr1) x) ((kl2 - kr1) kappas[f1] + (kl1 + kl2) kappas[f2])
     Conjugate[b[kl1, f1]] ** sigma[3] ** Conjugate[a[kl2, s1]] ** b[kr1, f2]) /
   (4 Sqrt[2] Sqrt[kl2] (kl1 + kl2) (kl2 - kr1) π^(3/2)) -
  (i e^(i (kl1+kl2-kr1) x) ((kl1 + kl2) kappas[f1] + (kl1 - kr1) kappas[f2])
     d[kr1, f1] ** sigma[3] ** Conjugate[a[kl1, s1]] ** Conjugate[d[kl2, f2]]) /
   (4 Sqrt[2] Sqrt[kl1] (kl1 + kl2) (kl1 - kr1) π^(3/2)) +
  (e^(-i (-kl1+kr1+kr2) x) ((kl1 - kr1) kappas[f1] + (kl1 - kr2) kappas[f2])
     d[kr2, f1] ** sigma[2] ** Conjugate[a[kl1, s1]] ** b[kr1, f2]) /
   (4 Sqrt[2] Sqrt[kl1] (kl1 - kr1) (kl1 - kr2) π^(3/2)) +
  (i e^(i (kl1-kr1-kr2) x) ((-kl1 + kr1) kappas[f1] + (kr1 + kr2) kappas[f2])
     d[kr2, f1] ** sigma[3] ** a[kr1, s1] ** Conjugate[d[kl1, f2]]) /
   (4 Sqrt[2] Sqrt[kr1] (-kl1 + kr1) (kr1 + kr2) π^(3/2)) -
  (e^(-i (kr1+kr2+kr3) x) ((kr1 + kr2) kappas[f1] + (kr2 + kr3) kappas[f2])
     d[kr3, f1] ** sigma[2] ** a[kr2, s1] ** b[kr1, f2]) /
   (4 Sqrt[2] Sqrt[kr2] (kr1 + kr2) (kr2 + kr3) π^(3/2)) +
  (e^(i (kl1+kl2-kr1) x) ((kl2 - kr1) kappaa[f1] + (kl1 - kr1) kappaa[f2]) Conjugate[b[kl1, f1]] **
     zeta[s1] ** sigma[2] ** a[kr1, s1] ** Conjugate[d[kl2, f2]]) /
   (4 Sqrt[2] Sqrt[kr1] (-kl1 + kr1) (-kl2 + kr1) π^(3/2)) -
  (e^(i (kl1+kl2+kl3) x) ((kl2 + kl3) kappaa[f1] + (kl1 + kl2) kappaa[f2]) Conjugate[b[kl1, f1]] **
     zeta[s1] ** sigma[2] ** Conjugate[a[kl2, s1]] ** Conjugate[d[kl3, f2]]) /
   (4 Sqrt[2] Sqrt[kl2] (kl1 + kl2) (kl2 + kl3) π^(3/2)) +
  (i e^(i (kl1-kr1-kr2) x) ((kr1 + kr2) kappaa[f1] + (-kl1 + kr2) kappaa[f2])
     Conjugate[b[kl1, f1]] ** zeta[s1] ** sigma[3] ** a[kr2, s1] ** b[kr1, f2]) /
   (4 Sqrt[2] Sqrt[kr2] (-kl1 + kr2) (kr1 + kr2) π^(3/2)) +
  (i e^(i (kl1+kl2-kr1) x) ((kl2 - kr1) kappaa[f1] + (kl1 + kl2) kappaa[f2])
     Conjugate[b[kl1, f1]] ** zeta[s1] ** sigma[3] ** Conjugate[a[kl2, s1]] **
     b[kr1, f2]) / (4 Sqrt[2] Sqrt[kl2] (kl1 + kl2) (kl2 - kr1) π^(3/2)) +
  (e^(-i (-kl1+kr1+kr2) x) ((-kl1 + kr1) kappaa[f1] + (-kl1 + kr2) kappaa[f2])
     d[kr2, f1] ** sigma[2] ** zeta[s1] ** Conjugate[a[kl1, s1]] ** b[kr1, f2]) /
   (4 Sqrt[2] Sqrt[kl1] (kl1 - kr1) (kl1 - kr2) π^(3/2)) -
  (e^(-i (kr1+kr2+kr3) x) ((kr1 + kr2) kappaa[f1] + (kr2 + kr3) kappaa[f2])
     d[kr3, f1] ** sigma[2] ** zeta[s1] ** a[kr2, s1] ** b[kr1, f2]) /
   (4 Sqrt[2] Sqrt[kr2] (kr1 + kr2) (kr2 + kr3) π^(3/2)) -
  (e^(i (kl1+kl2-kr1) x) ((kl1 + kl2) kappaa[f1] + (kl1 - kr1) kappaa[f2]) d[kr1, f1] ** sigma[1] **
     zeta[s1] ** sigma[2] ** Conjugate[a[kl1, s1]] ** Conjugate[d[kl2, f2]]) /
   (4 Sqrt[2] Sqrt[kl1] (kl1 + kl2) (kl1 - kr1) π^(3/2)) -
  (e^(i (kl1-kr1-kr2) x) ((-kl1 + kr1) kappaa[f1] + (kr1 + kr2) kappaa[f2]) d[kr2, f1] **
     sigma[1] ** zeta[s1] ** sigma[2] ** a[kr1, s1] ** Conjugate[d[kl1, f2]]) /
   (4 Sqrt[2] Sqrt[kr1] (-kl1 + kr1) (kr1 + kr2) π^(3/2))
```

In[187]:=

```
(*put in the coupling constant later*)
expr1baryon = relabel[Map[Distribute, qqbaryon[x, f1, f2] /. {kl1 → kkl1,
        kl2 → kkl2, kl3 → kkl3, kr1 → kkr1, kr2 → kkr2, kr3 → kkr3, s1 → ss1}] **
    Map[Distribute, qqbaryon[x, f2, f1]] //. prodexpand] //. prodfactor;
```

```
Out[186]= $Aborted
```

Since the delta[...] is removed in the following, the integration region can be complicated. expr2 contians all of the leading baryon interactions in the Hamiltonian.

```
(*gamma is the coupling constant*) expr2baryon =
 gamma / 2 * simpint[int[expr1baryon, {x}] //. intsumexpand /. Exp[x_] :> Exp[Factor[x]] /.
     int[Exp[a_ b_], {b_}] → 2 Pi delta[I a] /. delta[_?neverzero] → 0, k11]
```