■ Find chiral 4–fermioninteraction to leading order

We need the definition of xi from the Yukawa interactions

```
In[104]:= cxi[x_] = Conjugate[xi[x]] /. {Conjugate[x] → x,
            Conjugate[ka] → ka, Conjugate[kf] → kf, Conjugate[ka^n_] → ka^n} /.
          {ka → kka, kf → kks, rr → ll, sumrr → sumll} //. prodexpand //. sigmazeta
```

$$\text{Out[104]=}\ \frac{e^{-i\,kks\,x}\,muf\,d[kks]}{2\sqrt{\pi}}+\frac{e^{i\,kks\,x}\,muf\,Conjugate[b[kks]]\star\star sigma[1]}{2\sqrt{\pi}}-\frac{e^{-i\,kka\,x-i\,kks\,x}\,kappas\,d[kks]\star\star a[kka,\,sumll]}{4\sqrt{kka}\,\pi}-\frac{e^{i\,kka\,x-i\,kks\,x}\,kappas\,d[kks]\star\star Conjugate[a[kka,\,sumll]]}{4\sqrt{kka}\,\pi}-\frac{e^{-i\,kka\,x+i\,kks\,x}\,kappas\,Conjugate[b[kks]]\star\star sigma[1]\star\star a[kka,\,sumll]}{4\sqrt{kka}\,\pi}+\frac{e^{i\,kka\,x+i\,kks\,x}\,kappas\,Conjugate[b[kks]]\star\star sigma[1]\star\star Conjugate[a[kka,\,sumll]]}{4\sqrt{kka}\,\pi}-\frac{e^{-i\,kka\,x+i\,kks\,x}\,kappaa\,Conjugate[b[kks]]\star\star zeta[ll]\star\star sigma[1]\star\star a[kka,\,ll]}{4\sqrt{kka}\,\pi}+\frac{e^{i\,kka\,x+i\,kks\,x}\,kappaa\,Conjugate[b[kks]]\star\star zeta[ll]\star\star sigma[1]\star\star Conjugate[a[kka,\,ll]]}{4\sqrt{kka}\,\pi}-\frac{e^{-i\,kka\,x-i\,kks\,x}\,kappaa\,d[kks]\star\star sigma[1]\star\star zeta[ll]\star\star sigma[1]\star\star a[kka,\,ll]}{4\sqrt{kka}\,\pi}-\frac{e^{i\,kka\,x-i\,kks\,x}\,kappaa\,d[kks]\star\star sigma[1]\star\star zeta[ll]\star\star sigma[1]\star\star Conjugate[a[kka,\,ll]]}{4\sqrt{kka}\,\pi}$$

```
In[105]:= conju[x_] = Conjugate[u[x]] /. {Conjugate[x] → x, Conjugate[kf] → kf} /. kf → kks
```

$$\text{Out[105]=}\ \frac{e^{i\,kks\,x}\,Conjugate[b[kks]]+e^{-i\,kks\,x}\,d[kks]\star\star sigma[1]}{\sqrt{2\,\pi}}$$

The chiral four–fermioninteraction in the Hamiltonian is gamma qq**qq/2 where qq is:

In[106]:=

```
qq[x_] = Collect[relabel[Expand[I conju[x] ** sigma[2] ** Integrate[xi[x], x] -
           I Integrate[cxi[x], x] ** sigma[2] ** u[x] //. prodexpand //.
         sigmazeta]], {muf, kappas, kappaa}] //. prodfactor
```

Out[106]=

$$
\begin{aligned}
&\text{muf}\left(\frac{e^{i\,(kl1+kl2)\,x}\,(kl1+kl2)\,\text{Conjugate}[b[kl1]]\,**\,\text{sigma}[2]\,**\,\text{Conjugate}[d[kl2]]}{2\sqrt{2}\;kl1\;kl2\;\pi}+\right.\\
&\quad\frac{i\,e^{i\,(kl1-kr1)\,x}\,(kl1-kr1)\,\text{Conjugate}[b[kl1]]\,**\,\text{sigma}[3]\,**\,b[kr1]}{2\sqrt{2}\;kl1\;kr1\;\pi}-\\
&\quad\frac{i\,e^{i\,(kl1-kr1)\,x}\,(kl1-kr1)\,d[kr1]\,**\,\text{sigma}[3]\,**\,\text{Conjugate}[d[kl1]]}{2\sqrt{2}\;kl1\;kr1\;\pi}+\\
&\quad\left.\frac{e^{-i\,(kr1+kr2)\,x}\,(kr1+kr2)\,d[kr2]\,**\,\text{sigma}[2]\,**\,b[kr1]}{2\sqrt{2}\;kr1\;kr2\;\pi}\right)+\\
&\text{kappas}\left(-\left(e^{i\,(kl1+kl2-kr1)\,x}\,(kl1+kl2-2\,kr1)\,\text{Conjugate}[b[kl1]]\,**\,\text{sigma}[2]\,**\right.\right.\\
&\qquad a[kr1,\,s1]\,**\,\text{Conjugate}[d[kl2]]\Big)\Big/\left(4\sqrt{2}\,\sqrt{kr1}\,(-kl1+kr1)\,(-kl2+kr1)\,\pi^{3/2}\right)-\\
&\quad\left(e^{i\,(kl1+kl2+kl3)\,x}\,(kl1+2\,kl2+kl3)\,\text{Conjugate}[b[kl1]]\,**\,\text{sigma}[2]\,**\,\text{Conjugate}[\right.\\
&\qquad a[kl2,\,s1]]\,**\,\text{Conjugate}[d[kl3]]\Big)\Big/\left(4\sqrt{2}\,\sqrt{kl2}\,(kl1+kl2)\,(kl2+kl3)\,\pi^{3/2}\right)-\\
&\quad\left(i\,e^{i\,(kl1-kr1-kr2)\,x}\,(-kl1+kr1+2\,kr2)\,\text{Conjugate}[b[kl1]]\,**\,\text{sigma}[3]\,**\right.\\
&\qquad a[kr2,\,s1]\,**\,b[kr1]\Big)\Big/\left(4\sqrt{2}\,\sqrt{kr2}\,(-kl1+kr2)\,(kr1+kr2)\,\pi^{3/2}\right)+\\
&\quad\left(i\,e^{i\,(kl1+kl2-kr1)\,x}\,(kl1+2\,kl2-kr1)\,\text{Conjugate}[b[kl1]]\,**\,\text{sigma}[3]\,**\right.\\
&\qquad\text{Conjugate}[a[kl2,\,s1]]\,**\,b[kr1]\Big)\Big/\left(4\sqrt{2}\,\sqrt{kl2}\,(kl1+kl2)\,(kl2-kr1)\,\pi^{3/2}\right)-\\
&\quad\left(i\,e^{i\,(kl1+kl2-kr1)\,x}\,(2\,kl1+kl2-kr1)\,d[kr1]\,**\,\text{sigma}[3]\,**\,\text{Conjugate}[a[kl1,\,s1]]\,**\right.\\
&\qquad\text{Conjugate}[d[kl2]]\Big)\Big/\left(4\sqrt{2}\,\sqrt{kl1}\,(kl1+kl2)\,(kl1-kr1)\,\pi^{3/2}\right)+\\
&\quad\left(e^{-i\,(-kl1+kr1+kr2)\,x}\,(2\,kl1-kr1-kr2)\,d[kr2]\,**\,\text{sigma}[2]\,**\,\text{Conjugate}[a[kl1,\,s1]]\,**\right.\\
&\qquad b[kr1]\Big)\Big/\left(4\sqrt{2}\,\sqrt{kl1}\,(kl1-kr1)\,(kl1-kr2)\,\pi^{3/2}\right)+\\
&\quad\left(i\,e^{i\,(kl1-kr1-kr2)\,x}\,(-kl1+2\,kr1+kr2)\,d[kr2]\,**\,\text{sigma}[3]\,**\,a[kr1,\,s1]\,**\right.\\
&\qquad\text{Conjugate}[d[kl1]]\Big)\Big/\left(4\sqrt{2}\,\sqrt{kr1}\,(-kl1+kr1)\,(kr1+kr2)\,\pi^{3/2}\right)-\\
&\quad\left.\left.\frac{e^{-i\,(kr1+kr2+kr3)\,x}\,(kr1+2\,kr2+kr3)\,d[kr3]\,**\,\text{sigma}[2]\,**\,a[kr2,\,s1]\,**\,b[kr1]}{4\sqrt{2}\,\sqrt{kr2}\,(kr1+kr2)\,(kr2+kr3)\,\pi^{3/2}}\right)+\right.\\
&\text{kappaa}\left(\left(e^{i\,(kl1+kl2-kr1)\,x}\,(kl1+kl2-2\,kr1)\,\text{Conjugate}[b[kl1]]\,**\,\text{zeta}[s1]\,**\,\text{sigma}[2]\,**\right.\right.\\
&\qquad a[kr1,\,s1]\,**\,\text{Conjugate}[d[kl2]]\Big)\Big/\left(4\sqrt{2}\,\sqrt{kr1}\,(-kl1+kr1)\,(-kl2+kr1)\,\pi^{3/2}\right)-\\
&\quad\left(e^{i\,(kl1+kl2+kl3)\,x}\,(kl1+2\,kl2+kl3)\,\text{Conjugate}[b[kl1]]\,**\,\text{zeta}[s1]\,**\right.\\
&\qquad\text{sigma}[2]\,**\,\text{Conjugate}[a[kl2,\,s1]]\,**\,\text{Conjugate}[d[kl3]]\Big)\Big/\\
&\qquad\left(4\sqrt{2}\,\sqrt{kl2}\,(kl1+kl2)\,(kl2+kl3)\,\pi^{3/2}\right)+\left(i\,e^{i\,(kl1-kr1-kr2)\,x}\,(-kl1+kr1+2\,kr2)\right.\\
&\qquad\text{Conjugate}[b[kl1]]\,**\,\text{zeta}[s1]\,**\,\text{sigma}[3]\,**\,a[kr2,\,s1]\,**\,b[kr1]\Big)\Big/\\
&\qquad\left(4\sqrt{2}\,\sqrt{kr2}\,(-kl1+kr2)\,(kr1+kr2)\,\pi^{3/2}\right)+\left(i\,e^{i\,(kl1+kl2-kr1)\,x}\,(kl1+2\,kl2-kr1)\right.\\
&\qquad\text{Conjugate}[b[kl1]]\,**\,\text{zeta}[s1]\,**\,\text{sigma}[3]\,**\,\text{Conjugate}[a[kl2,\,s1]]\,**\,b[kr1]\Big)\Big/\\
&\qquad\left(4\sqrt{2}\,\sqrt{kl2}\,(kl1+kl2)\,(kl2-kr1)\,\pi^{3/2}\right)+\left(e^{-i\,(-kl1+kr1+kr2)\,x}\,(-2\,kl1+kr1+kr2)\right.\\
&\qquad d[kr2]\,**\,\text{sigma}[2]\,**\,\text{zeta}[s1]\,**\,\text{Conjugate}[a[kl1,\,s1]]\,**\,b[kr1]\Big)\Big/\\
&\qquad\left(4\sqrt{2}\,\sqrt{kl1}\,(kl1-kr1)\,(kl1-kr2)\,\pi^{3/2}\right)-\left(e^{-i\,(kr1+kr2+kr3)\,x}\,(kr1+2\,kr2+kr3)\right.\\
&\qquad d[kr3]\,**\,\text{sigma}[2]\,**\,\text{zeta}[s1]\,**\,a[kr2,\,s1]\,**\,b[kr1]\Big)\Big/\\
&\qquad\left(4\sqrt{2}\,\sqrt{kr2}\,(kr1+kr2)\,(kr2+kr3)\,\pi^{3/2}\right)-\left(e^{i\,(kl1+kl2-kr1)\,x}\,(2\,kl1+kl2-kr1)\,d[kr1]\,**\right.\\
&\qquad\text{sigma}[1]\,**\,\text{zeta}[s1]\,**\,\text{sigma}[2]\,**\,\text{Conjugate}[a[kl1,\,s1]]\,**\,\text{Conjugate}[d[kl2]]\Big)\Big/\\
&\qquad\left(4\sqrt{2}\,\sqrt{kl1}\,(kl1+kl2)\,(kl1-kr1)\,\pi^{3/2}\right)-\left(e^{i\,(kl1-kr1-kr2)\,x}\,(-kl1+2\,kr1+kr2)\right.\\
&\qquad d[kr2]\,**\,\text{sigma}[1]\,**\,\text{zeta}[s1]\,**\,\text{sigma}[2]\,**\,a[kr1,\,s1]\,**\,\text{Conjugate}[d[kl1]]\Big)\Big/\\
&\qquad\left.\left.\left(4\sqrt{2}\,\sqrt{kr1}\,(-kl1+kr1)\,(kr1+kr2)\,\pi^{3/2}\right)\right)\right)
\end{aligned}
$$

■ Select terms that are leading order in N for mesons ("pinching" terms):

```
In[110]:= qqpinch = qq[x] /.
            a_NonCommutativeMultiply :> Block[{non = a //. {sigma[_] ** c_ → c, zeta[_] ** c_ → c}},
              0 /; Count[non, Conjugate[_]] != Length[non] && Count[non, Conjugate[_]] != 0]
```

$$\begin{aligned}\text{Out[110]=}\ & \text{muf}\left(\frac{e^{i\,(kl1+kl2)\,x}\ (kl1+kl2)\ \text{Conjugate}[b[kl1]]\ **\ \text{sigma}[2]\ **\ \text{Conjugate}[d[kl2]]}{2\sqrt{2}\ kl1\ kl2\ \pi}+\right.\\ & \left.\frac{e^{-i\,(kr1+kr2)\,x}\ (kr1+kr2)\ d[kr2]\ **\ \text{sigma}[2]\ **\ b[kr1]}{2\sqrt{2}\ kr1\ kr2\ \pi}\right)+\\ & \text{kappas}\left(-\left(e^{i\,(kl1+kl2+kl3)\,x}\ (kl1+2\,kl2+kl3)\ \text{Conjugate}[b[kl1]]\ **\ \text{sigma}[2]\ **\ \text{Conjugate}[\right.\right.\\ & \left.\left.a[kl2,\ s1]]\ **\ \text{Conjugate}[d[kl3]]\right)\Big/\left(4\sqrt{2}\ \sqrt{kl2}\ (kl1+kl2)\ (kl2+kl3)\ \pi^{3/2}\right)-\\ & \left.\frac{e^{-i\,(kr1+kr2+kr3)\,x}\ (kr1+2\,kr2+kr3)\ d[kr3]\ **\ \text{sigma}[2]\ **\ a[kr2,\ s1]\ **\ b[kr1]}{4\sqrt{2}\ \sqrt{kr2}\ (kr1+kr2)\ (kr2+kr3)\ \pi^{3/2}}\right)+\\ & \text{kappaa}\left(-\left(e^{i\,(kl1+kl2+kl3)\,x}\ (kl1+2\,kl2+kl3)\ \text{Conjugate}[b[kl1]]\ **\ \text{zeta}[s1]\ **\right.\right.\\ & \left.\text{sigma}[2]\ **\ \text{Conjugate}[a[kl2,\ s1]]\ **\ \text{Conjugate}[d[kl3]]\right)\Big/\\ & \left(4\sqrt{2}\ \sqrt{kl2}\ (kl1+kl2)\ (kl2+kl3)\ \pi^{3/2}\right)-\left(e^{-i\,(kr1+kr2+kr3)\,x}\ (kr1+2\,kr2+kr3)\right.\\ & \left.d[kr3]\ **\ \text{sigma}[2]\ **\ \text{zeta}[s1]\ **\ a[kr2,\ s1]\ **\ b[kr1]\right)\Big/\\ & \left.\left(4\sqrt{2}\ \sqrt{kr2}\ (kr1+kr2)\ (kr2+kr3)\ \pi^{3/2}\right)\right)\end{aligned}$$

```
In[111]:= (*put in the coupling constant later*)
          expr1pinch = Collect[relabel[Map[Distribute, qqpinch /. {kl1 → kkl1,
                kl2 → kkl2, kl3 → kkl3, kr1 → kkr1, kr2 → kkr2, kr3 → kkr3, s1 → ss1}] **
              Map[Distribute, qqpinch] //. prodexpand], {muf, kappaa, kappas}] //. prodfactor;
```

expr2 contians all of the "pinching" interactions in the Hamiltonian.

```
In[112]:= (*gamma is the coupling constant*)expr2pinch =
           gamma / 2 * simpint[int[expr1pinch, {x}] //. intsumexpand /. Exp[x_] :> Exp[Factor[x]] /.
               int[Exp[a_ b_], {b_}] → 2 Pi delta[I a] /. delta[_?neverzero] → 0, kl1]
```

$$\begin{aligned}\text{Out[112]=}\ & \frac{1}{2}\ \text{gamma}\\ & \left(\text{muf}^2\left(\frac{1}{4\ kl1\ kl2\ kr1\ kr2\ \pi}\left((kl1+kl2)\ (kr1+kr2)\ \text{delta}[-kl1-kl2+kr1+kr2]\ \text{Conjugate}[\right.\right.\right.\\ & \left.\left.b[kl1]]\ **\ \text{sigma}[2]\ **\ \text{Conjugate}[d[kl2]]\ **\ d[kr2]\ **\ \text{sigma}[2]\ **\ b[kr1]\right)+\\ & \frac{1}{4\ kl1\ kl2\ kr1\ kr2\ \pi}\left((kl1+kl2)\ (kr1+kr2)\ \text{delta}[-kl1-kl2+kr1+kr2]\ d[kr2]\ **\right.\\ & \left.\left.\text{sigma}[2]\ **\ b[kr1]\ **\ \text{Conjugate}[b[kl1]]\ **\ \text{sigma}[2]\ **\ \text{Conjugate}[d[kl2]]\right)\right)+\\ & \text{kappas}^2\left(\left((kl1+2\,kl2+kl3)\ (kr1+2\,kr2+kr3)\ \text{delta}[-kl1-kl2-kl3+kr1+kr2+kr3]\right.\right.\\ & \text{Conjugate}[b[kl1]]\ **\ \text{sigma}[2]\ **\ \text{Conjugate}[a[kl2,\ s2]]\ **\\ & \left.\text{Conjugate}[d[kl3]]\ **\ d[kr3]\ **\ \text{sigma}[2]\ **\ a[kr2,\ s1]\ **\ b[kr1]\right)\Big/\\ & \left(16\sqrt{kl2}\ (kl1+kl2)\ (kl2+kl3)\ \sqrt{kr2}\ (kr1+kr2)\ (kr2+kr3)\ \pi^2\right)+\\ & \left((kl1+2\,kl2+kl3)\ (kr1+2\,kr2+kr3)\ \text{delta}[-kl1-kl2-kl3+kr1+kr2+kr3]\right.\\ & d[kr3]\ **\ \text{sigma}[2]\ **\ a[kr2,\ s2]\ **\ b[kr1]\ **\ \text{Conjugate}[b[kl1]]\ **\\ & \left.\left.\text{sigma}[2]\ **\ \text{Conjugate}[a[kl2,\ s1]]\ **\ \text{Conjugate}[d[kl3]]\right)\Big/\right.\\ & \left.\left(16\sqrt{kl2}\ (kl1+kl2)\ (kl2+kl3)\ \sqrt{kr2}\ (kr1+kr2)\ (kr2+kr3)\ \pi^2\right)\right)+\text{muf}\\ & \left(\text{kappas}\left(-\left((kl1+2\,kl2+kl3)\ (kr1+kr2)\ \text{delta}[-kl1-kl2-kl3+kr1+kr2]\ \text{Conjugate}[\right.\right.\right.\\ & \left.\left.b[kl1]]\ **\ \text{sigma}[2]\ **\ \text{Conjugate}[a[kl2,\ s1]]\ **\ \text{Conjugate}[d[kl3]]\ **\right.\right.\\ & \left.\left.d[kr2]\ **\ \text{sigma}[2]\ **\ b[kr1]\right)\Big/\left(8\sqrt{kl2}\ (kl1+kl2)\ (kl2+kl3)\ kr1\ kr2\ \pi^{3/2}\right)-\right.\\ & \left((kl1+kl2)\ (kr1+2\,kr2+kr3)\ \text{delta}[-kl1-kl2+kr1+kr2+kr3]\right.\\ & \text{Conjugate}[b[kl1]]\ **\ \text{sigma}[2]\ **\ \text{Conjugate}[d[kl2]]\ **\ d[kr3]\ **\ \text{sigma}[2]\ **\\ & \left.\left.a[kr2,\ s1]\ **\ b[kr1]\right)\Big/\left(8\ kl1\ kl2\ \sqrt{kr2}\ (kr1+kr2)\ (kr2+kr3)\ \pi^{3/2}\right)-\right.\end{aligned}$$

$$((kl1 + 2\,kl2 + kl3)\,(kr1 + kr2)\,\text{delta}[-kl1 - kl2 - kl3 + kr1 + kr2]\,d[kr2] ** \text{sigma}[2] ** b[kr1] ** \text{Conjugate}[b[kl1]] ** \text{sigma}[2] ** \text{Conjugate}[a[kl2, s1]] ** \text{Conjugate}[d[kl3]]) \Big/ \left(8\sqrt{kl2}\,(kl1 + kl2)\,(kl2 + kl3)\,kr1\,kr2\,\pi^{3/2}\right) -$$
$$((kl1 + kl2)\,(kr1 + 2\,kr2 + kr3)\,\text{delta}[-kl1 - kl2 + kr1 + kr2 + kr3]\,d[kr3] ** \text{sigma}[2] ** a[kr2, s1] ** b[kr1] ** \text{Conjugate}[b[kl1]] ** \text{sigma}[2] ** \text{Conjugate}[d[kl2]]) \Big/ \left(8\,kl1\,kl2\sqrt{kr2}\,(kr1 + kr2)\,(kr2 + kr3)\,\pi^{3/2}\right)) +$$
$$\text{kappaa}\left(-((kl1 + kl2)\,(kr1 + 2\,kr2 + kr3)\,\text{delta}[-kl1 - kl2 + kr1 + kr2 + kr3]\,\text{Conjugate}[b[kl1]] ** \text{sigma}[2] ** \text{Conjugate}[d[kl2]] ** d[kr3] ** \text{sigma}[2] ** \text{zeta}[s1] ** a[kr2, s1] ** b[kr1]) \Big/ \left(8\,kl1\,kl2\sqrt{kr2}\,(kr1 + kr2)\,(kr2 + kr3)\,\pi^{3/2}\right) - ((kl1 + 2\,kl2 + kl3)\,(kr1 + kr2)\,\text{delta}[-kl1 - kl2 - kl3 + kr1 + kr2]\,\text{Conjugate}[b[kl1]] ** \text{zeta}[s1] ** \text{sigma}[2] ** \text{Conjugate}[a[kl2, s1]] ** \text{Conjugate}[d[kl3]] ** d[kr2] ** \text{sigma}[2] ** b[kr1]) \Big/ \left(8\sqrt{kl2}\,(kl1 + kl2)\,(kl2 + kl3)\,kr1\,kr2\,\pi^{3/2}\right) -$$
$$((kl1 + 2\,kl2 + kl3)\,(kr1 + kr2)\,\text{delta}[-kl1 - kl2 - kl3 + kr1 + kr2]\,d[kr2] ** \text{sigma}[2] ** b[kr1] ** \text{Conjugate}[b[kl1]] ** \text{zeta}[s1] ** \text{sigma}[2] ** \text{Conjugate}[a[kl2, s1]] ** \text{Conjugate}[d[kl3]]) \Big/ \left(8\sqrt{kl2}\,(kl1 + kl2)\,(kl2 + kl3)\,kr1\,kr2\,\pi^{3/2}\right) -$$
$$((kl1 + 2\,kl2 + kl3)\,(kr1 + 2\,kr2 + kr3)\,\text{delta}[-kl1 - kl2 + kr1 + kr2 + kr3]\,d[kr3] ** \text{sigma}[2] ** \text{zeta}[s1] ** a[kr2, s1] ** b[kr1] ** \text{Conjugate}[b[kl1]] ** \text{sigma}[2] ** \text{Conjugate}[a[kl2, s1]] ** \text{Conjugate}[d[kl3]]) \Big/ \left(8\,kl1\,kl2\sqrt{kr2}\,(kr1 + kr2)\,(kr2 + kr3)\,\pi^{3/2}\right)) + \text{kappaa}\,\text{kappas}$$
$$\Big(((kl1 + 2\,kl2 + kl3)\,(kr1 + 2\,kr2 + kr3)\,\text{delta}[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3]\,\text{Conjugate}[b[kl1]] ** \text{sigma}[2] ** \text{Conjugate}[a[kl2, s2]] ** \text{Conjugate}[d[kl3]] ** d[kr3] ** \text{sigma}[2] ** \text{zeta}[s1] ** a[kr2, s1] ** b[kr1]) \Big/ \left(16\sqrt{kl2}\,(kl1 + kl2)\,(kl2 + kl3)\sqrt{kr2}\,(kr1 + kr2)\,(kr2 + kr3)\,\pi^2\right) +$$
$$((kl1 + 2\,kl2 + kl3)\,(kr1 + 2\,kr2 + kr3)\,\text{delta}[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3]\,\text{Conjugate}[b[kl1]] ** \text{zeta}[s2] ** \text{sigma}[2] ** \text{Conjugate}[a[kl2, s2]] ** \text{Conjugate}[d[kl3]] ** d[kr3] ** \text{sigma}[2] ** a[kr2, s1] ** b[kr1]) \Big/ \left(16\sqrt{kl2}\,(kl1 + kl2)\,(kl2 + kl3)\sqrt{kr2}\,(kr1 + kr2)\,(kr2 + kr3)\,\pi^2\right) +$$
$$((kl1 + 2\,kl2 + kl3)\,(kr1 + 2\,kr2 + kr3)\,\text{delta}[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3]\,d[kr3] ** \text{sigma}[2] ** a[kr2, s2] ** b[kr1] ** \text{Conjugate}[b[kl1]] ** \text{zeta}[s1] ** \text{sigma}[2] ** \text{Conjugate}[a[kl2, s1]] ** \text{Conjugate}[d[kl3]]) \Big/ \left(16\sqrt{kl2}\,(kl1 + kl2)\,(kl2 + kl3)\sqrt{kr2}\,(kr1 + kr2)\,(kr2 + kr3)\,\pi^2\right) +$$
$$((kl1 + 2\,kl2 + kl3)\,(kr1 + 2\,kr2 + kr3)\,\text{delta}[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3]\,d[kr3] ** \text{sigma}[2] ** \text{zeta}[s2] ** a[kr2, s2] ** b[kr1] ** \text{Conjugate}[b[kl1]] ** \text{sigma}[2] ** \text{Conjugate}[a[kl2, s1]] ** \text{Conjugate}[d[kl3]]) \Big/ \left(16\sqrt{kl2}\,(kl1 + kl2)\,(kl2 + kl3)\sqrt{kr2}\,(kr1 + kr2)\,(kr2 + kr3)\,\pi^2\right)) +$$
$$\text{kappaa}^2\left(((kl1 + 2\,kl2 + kl3)\,(kr1 + 2\,kr2 + kr3)\,\text{delta}[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3]\,\text{Conjugate}[b[kl1]] ** \text{zeta}[s2] ** \text{sigma}[2] ** \text{Conjugate}[a[kl2, s2]] ** \text{Conjugate}[d[kl3]] ** d[kr3] ** \text{sigma}[2] ** \text{zeta}[s1] ** a[kr2, s1] ** b[kr1]) \Big/ \left(16\sqrt{kl2}\,(kl1 + kl2)\,(kl2 + kl3)\sqrt{kr2}\,(kr1 + kr2)\,(kr2 + kr3)\,\pi^2\right) +$$
$$((kl1 + 2\,kl2 + kl3)\,(kr1 + 2\,kr2 + kr3)\,\text{delta}[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3]\,d[kr3] ** \text{sigma}[2] ** \text{zeta}[s2] ** a[kr2, s2] ** b[kr1] ** \text{Conjugate}[b[kl1]] ** \text{sigma}[2] ** \text{Conjugate}[a[kl2, s1]] ** \text{Conjugate}[d[kl3]]) \Big/ \left(16\sqrt{kl2}\,(kl1 + kl2)\,(kl2 + kl3)\sqrt{kr2}\,(kr1 + kr2)\,(kr2 + kr3)\,\pi^2\right)\Big)$$

■ Select terms that act on a baryon (no anti−quarks)

In[107]:=
```
qqbaryon = qq[x] /. d[_] → 0 /. {0 ** a_ → 0, a_ ** 0 → 0}
```

```
Out[107]= (i E^(i (kl1-kr1) x) (kl1 - kr1) muf Conjugate[b[kl1]] ** sigma[3] ** b[kr1]) /
            (2 Sqrt[2] kl1 kr1 Pi) +
          kappas (-(i E^(i (kl1-kr1-kr2) x) (-kl1 + kr1 + 2 kr2) Conjugate[b[kl1]] ** sigma[3] **
                a[kr2, s1] ** b[kr1]) / (4 Sqrt[2] Sqrt[kr2] (-kl1 + kr2) (kr1 + kr2) Pi^(3/2)) +
             (i E^(i (kl1+kl2-kr1) x) (kl1 + 2 kl2 - kr1) Conjugate[b[kl1]] ** sigma[3] **
                Conjugate[a[kl2, s1]] ** b[kr1]) / (4 Sqrt[2] Sqrt[kl2] (kl1 + kl2) (kl2 - kr1) Pi^(3/2))) +
          kappaa ((i E^(i (kl1-kr1-kr2) x) (-kl1 + kr1 + 2 kr2) Conjugate[b[kl1]] ** zeta[s1] **
                sigma[3] ** a[kr2, s1] ** b[kr1]) / (4 Sqrt[2] Sqrt[kr2] (-kl1 + kr2) (kr1 + kr2) Pi^(3/2)) +
             (i E^(i (kl1+kl2-kr1) x) (kl1 + 2 kl2 - kr1) Conjugate[b[kl1]] ** zeta[s1] ** sigma[3] **
                Conjugate[a[kl2, s1]] ** b[kr1]) / (4 Sqrt[2] Sqrt[kl2] (kl1 + kl2) (kl2 - kr1) Pi^(3/2)))
```

```
In[108]:= (*put in the coupling constant later*)expr1baryon =
           Collect[relabel[Map[Distribute, qqbaryon /. {kl1 → kk1l, kl2 → kk2l, kl3 → kk3l,
                   kr1 → kkr1, kr2 → kkr2, kr3 → kkr3, s1 → ss1}] ** Map[Distribute, qqbaryon] //.
               prodexpand], {muf, kappaa, kappas}] //. prodfactor;
```

Since the delta[...] is removed in the following, the integration region can be complicated. expr2 contians all of the leading baryon interactions in the Hamiltonian.

```
In[109]:= (*gamma is the coupling constant*) expr2baryon =
           gamma / 2 * simpint[int[expr1baryon, {x}] //. intsumexpand /. Exp[x_] :→ Exp[Factor[x]] /.
               int[Exp[a_ b_], {b_}] → 2 Pi delta[I a] /. delta[_?neverzero] → 0, kl1]
```

```
Out[109]= (1/2) gamma
          (-(1/(4 kl1 kl2 kr1 kr2 Pi)) ((kl2 - kr1) (kl1 - kr2) muf^2 delta[-kl1 - kl2 + kr1 + kr2] Conjugate[
                b[kl1]] ** sigma[3] ** b[kr2] ** Conjugate[b[kl2]] ** sigma[3] ** b[kr1]) +
            kappas^2 (((kl2 + 2 kl3 - kr1) (-kl1 + kr2 + 2 kr3) delta[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3]
                  Conjugate[b[kl1]] ** sigma[3] ** a[kr3, s2] ** b[kr2] **
                  Conjugate[b[kl2]] ** sigma[3] ** Conjugate[a[kl3, s1]] ** b[kr1]) /
                (16 Sqrt[kl3] (kl2 + kl3) (kl3 - kr1) Sqrt[kr3] (-kl1 + kr3) (kr2 + kr3) Pi^2) -
               ((-kl2 + kr1 + 2 kr2) (-kl1 + kr3 + 2 kr4) delta[-kl1 - kl2 + kr1 + kr2 + kr3 + kr4]
                  Conjugate[b[kl1]] ** sigma[3] ** a[kr4, s2] ** b[kr3] **
                  Conjugate[b[kl2]] ** sigma[3] ** a[kr2, s1] ** b[kr1]) /
                (16 Sqrt[kr2] (-kl2 + kr2) (kr1 + kr2) Sqrt[kr4] (-kl1 + kr4) (kr3 + kr4) Pi^2) -
               ((kl3 + 2 kl4 - kr1) (kl1 + 2 kl2 - kr2) delta[-kl1 - kl2 - kl3 - kl4 + kr1 + kr2]
                  Conjugate[b[kl1]] ** sigma[3] ** Conjugate[a[kl2, s2]] ** b[kr2] **
                  Conjugate[b[kl3]] ** sigma[3] ** Conjugate[a[kl4, s1]] ** b[kr1]) /
                (16 Sqrt[kl2] (kl1 + kl2) Sqrt[kl4] (kl3 + kl4) (kl4 - kr1) (kl2 - kr2) Pi^2) +
               ((-kl3 + kr1 + 2 kr2) (kl1 + 2 kl2 - kr3) delta[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3]
                  Conjugate[b[kl1]] ** sigma[3] ** Conjugate[a[kl2, s2]] **
                  b[kr3] ** Conjugate[b[kl3]] ** sigma[3] ** a[kr2, s1] ** b[kr1]) /
                (16 Sqrt[kl2] (kl1 + kl2) Sqrt[kr2] (-kl3 + kr2) (kr1 + kr2) (kl2 - kr3) Pi^2)) +
            muf (kappas (((kl2 - kr1) (-kl1 + kr2 + 2 kr3) delta[-kl1 - kl2 + kr1 + kr2 + kr3]
                  Conjugate[b[kl1]] ** sigma[3] ** a[kr3, s1] ** b[kr2] ** Conjugate[b[kl2]] **
                  sigma[3] ** b[kr1]) / (8 kl2 kr1 Sqrt[kr3] (-kl1 + kr3) (kr2 + kr3) Pi^(3/2)) -
               ((kl2 + 2 kl3 - kr1) (kl1 - kr2) delta[-kl1 - kl2 - kl3 + kr1 + kr2]
                  Conjugate[b[kl1]] ** sigma[3] ** b[kr2] ** Conjugate[b[kl2]] ** sigma[3] **
                  Conjugate[a[kl3, s1]] ** b[kr1]) / (8 kl1 Sqrt[kl3] (kl2 + kl3) (kl3 - kr1) kr2
                Pi^(3/2)) + ((-kl2 + kr1 + 2 kr2) (kl1 - kr3) delta[-kl1 - kl2 + kr1 + kr2 + kr3]
                Conjugate[b[kl1]] ** sigma[3] ** b[kr3] ** Conjugate[b[kl2]] ** sigma[3] **
                  a[kr2, s1] ** b[kr1]) / (8 kl1 Sqrt[kr2] (-kl2 + kr2) (kr1 + kr2) kr3 Pi^(3/2)) -
               ((kl3 - kr1) (kl1 + 2 kl2 - kr2) delta[-kl1 - kl2 - kl3 + kr1 + kr2]
                  Conjugate[b[kl1]] ** sigma[3] ** Conjugate[a[kl2, s1]] **
```

```
        b[kr2] ** Conjugate[b[kl3]] ** sigma[3] ** b[kr1]) /
      (8 Sqrt[kl2] (kl1 + kl2) kl3 kr1 (kl2 - kr2) Pi^(3/2))) +
  kappaa (-((kl2 + 2 kl3 - kr1) (kl1 - kr2) delta[-kl1 - kl2 - kl3 + kr1 + kr2]
       Conjugate[b[kl1]] ** sigma[3] ** b[kr2] ** Conjugate[b[kl2]] **
        zeta[s1] ** sigma[3] ** Conjugate[a[kl3, s1]] ** b[kr1]) /
      (8 kl1 Sqrt[kl3] (kl2 + kl3) (kl3 - kr1) kr2 Pi^(3/2)) - ((-kl2 + kr1 + 2 kr2) (kl1 - kr3)
       delta[-kl1 - kl2 + kr1 + kr2 + kr3] Conjugate[b[kl1]] ** sigma[3] ** b[kr3] **
        Conjugate[b[kl2]] ** zeta[s1] ** sigma[3] ** a[kr2, s1] ** b[kr1]) /
      (8 kl1 Sqrt[kr2] (-kl2 + kr2) (kr1 + kr2) kr3 Pi^(3/2)) - ((kl2 - kr1) (-kl1 + kr2 + 2 kr3)
       delta[-kl1 - kl2 + kr1 + kr2 + kr3] Conjugate[b[kl1]] ** zeta[s1] ** sigma[3] **
        a[kr3, s1] ** b[kr2] ** Conjugate[b[kl2]] ** sigma[3] ** b[kr1]) /
      (8 kl2 kr1 Sqrt[kr3] (-kl1 + kr3) (kr2 + kr3) Pi^(3/2)) -
     ((kl3 - kr1) (kl1 + 2 kl2 - kr2) delta[-kl1 - kl2 - kl3 + kr1 + kr2]
       Conjugate[b[kl1]] ** zeta[s1] ** sigma[3] ** Conjugate[a[kl2, s1]] **
        b[kr2] ** Conjugate[b[kl3]] ** sigma[3] ** b[kr1]) /
      (8 Sqrt[kl2] (kl1 + kl2) kl3 kr1 (kl2 - kr2) Pi^(3/2)))) + kappaa kappas
 (((kl2 + 2 kl3 - kr1) (-kl1 + kr2 + 2 kr3) delta[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3]
     Conjugate[b[kl1]] ** sigma[3] ** a[kr3, s2] ** b[kr2] ** Conjugate[b[kl2]] **
      zeta[s1] ** sigma[3] ** Conjugate[a[kl3, s1]] ** b[kr1]) /
    (16 Sqrt[kl3] (kl2 + kl3) (kl3 - kr1) Sqrt[kr3] (-kl1 + kr3) (kr2 + kr3) Pi^2) +
   ((-kl2 + kr1 + 2 kr2) (-kl1 + kr3 + 2 kr4) delta[-kl1 - kl2 + kr1 + kr2 + kr3 + kr4]
     Conjugate[b[kl1]] ** sigma[3] ** a[kr4, s2] ** b[kr3] **
      Conjugate[b[kl2]] ** zeta[s1] ** sigma[3] ** a[kr2, s1] ** b[kr1]) /
    (16 Sqrt[kr2] (-kl2 + kr2) (kr1 + kr2) Sqrt[kr4] (-kl1 + kr4) (kr3 + kr4) Pi^2) -
   ((kl3 + 2 kl4 - kr1) (kl1 + 2 kl2 - kr2) delta[-kl1 - kl2 - kl3 - kl4 + kr1 + kr2]
     Conjugate[b[kl1]] ** sigma[3] ** Conjugate[a[kl2, s2]] ** b[kr2] **
      Conjugate[b[kl3]] ** zeta[s1] ** sigma[3] ** Conjugate[a[kl4, s1]] ** b[kr1]) /
    (16 Sqrt[kl2] (kl1 + kl2) Sqrt[kl4] (kl3 + kl4) (kl4 - kr1) (kl2 - kr2) Pi^2) -
   ((-kl3 + kr1 + 2 kr2) (kl1 + 2 kl2 - kr3) delta[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3]
     Conjugate[b[kl1]] ** sigma[3] ** Conjugate[a[kl2, s2]] ** b[kr3] **
      Conjugate[b[kl3]] ** zeta[s1] ** sigma[3] ** a[kr2, s1] ** b[kr1]) /
    (16 Sqrt[kl2] (kl1 + kl2) Sqrt[kr2] (-kl3 + kr2) (kr1 + kr2) (kl2 - kr3) Pi^2) -
   ((kl2 + 2 kl3 - kr1) (-kl1 + kr2 + 2 kr3) delta[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3]
     Conjugate[b[kl1]] ** zeta[s2] ** sigma[3] ** a[kr3, s2] ** b[kr2] **
      Conjugate[b[kl2]] ** sigma[3] ** Conjugate[a[kl3, s1]] ** b[kr1]) /
    (16 Sqrt[kl3] (kl2 + kl3) (kl3 - kr1) Sqrt[kr3] (-kl1 + kr3) (kr2 + kr3) Pi^2) +
   ((-kl2 + kr1 + 2 kr2) (-kl1 + kr3 + 2 kr4) delta[-kl1 - kl2 + kr1 + kr2 + kr3 + kr4]
     Conjugate[b[kl1]] ** zeta[s2] ** sigma[3] ** a[kr4, s2] **
      b[kr3] ** Conjugate[b[kl2]] ** sigma[3] ** a[kr2, s1] ** b[kr1]) /
    (16 Sqrt[kr2] (-kl2 + kr2) (kr1 + kr2) Sqrt[kr4] (-kl1 + kr4) (kr3 + kr4) Pi^2) -
   ((kl3 + 2 kl4 - kr1) (kl1 + 2 kl2 - kr2) delta[-kl1 - kl2 - kl3 - kl4 + kr1 + kr2]
     Conjugate[b[kl1]] ** zeta[s2] ** sigma[3] ** Conjugate[a[kl2, s2]] **
      b[kr2] ** Conjugate[b[kl3]] ** sigma[3] ** Conjugate[a[kl4, s1]] ** b[kr1]) /
    (16 Sqrt[kl2] (kl1 + kl2) Sqrt[kl4] (kl3 + kl4) (kl4 - kr1) (kl2 - kr2) Pi^2) +
   ((-kl3 + kr1 + 2 kr2) (kl1 + 2 kl2 - kr3) delta[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3]
     Conjugate[b[kl1]] ** zeta[s2] ** sigma[3] ** Conjugate[a[kl2, s2]] **
      b[kr3] ** Conjugate[b[kl3]] ** sigma[3] ** a[kr2, s1] ** b[kr1]) /
    (16 Sqrt[kl2] (kl1 + kl2) Sqrt[kr2] (-kl3 + kr2) (kr1 + kr2) (kl2 - kr3) Pi^2)) +
 kappaa^2 (-((kl2 + 2 kl3 - kr1) (-kl1 + kr2 + 2 kr3) delta[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3]
       Conjugate[b[kl1]] ** zeta[s2] ** sigma[3] ** a[kr3, s2] ** b[kr2] ** Conjugate[
        b[kl2]] ** zeta[s1] ** sigma[3] ** Conjugate[a[kl3, s1]] ** b[kr1]) /
      (16 Sqrt[kl3] (kl2 + kl3) (kl3 - kr1) Sqrt[kr3] (-kl1 + kr3) (kr2 + kr3) Pi^2) -
```

```
((-kl2 + kr1 + 2 kr2) (-kl1 + kr3 + 2 kr4) delta[-kl1 - kl2 + kr1 + kr2 + kr3 + kr4]
   Conjugate[b[kl1]] ** zeta[s2] ** sigma[3] ** a[kr4, s2] ** b[kr3] **
    Conjugate[b[kl2]] ** zeta[s1] ** sigma[3] ** a[kr2, s1] ** b[kr1]) /
 (16 Sqrt[kr2] (-kl2 + kr2) (kr1 + kr2) Sqrt[kr4] (-kl1 + kr4) (kr3 + kr4) π^2) -
((kl3 + 2 kl4 - kr1) (kl1 + 2 kl2 - kr2) delta[-kl1 - kl2 - kl3 - kl4 + kr1 + kr2]
   Conjugate[b[kl1]] ** zeta[s2] ** sigma[3] ** Conjugate[a[kl2, s2]] ** b[kr2] **
    Conjugate[b[kl3]] ** zeta[s1] ** sigma[3] ** Conjugate[a[kl4, s1]] ** b[kr1]) /
 (16 Sqrt[kl2] (kl1 + kl2) Sqrt[kl4] (kl3 + kl4) (kl4 - kr1) (kl2 - kr2) π^2) -
((-kl3 + kr1 + 2 kr2) (kl1 + 2 kl2 - kr3) delta[-kl1 - kl2 - kl3 + kr1 + kr2 + kr3]
   Conjugate[b[kl1]] ** zeta[s2] ** sigma[3] ** Conjugate[a[kl2, s2]] **
    b[kr3] ** Conjugate[b[kl3]] ** zeta[s1] ** sigma[3] ** a[kr2, s1] ** b[kr1]) /
 (16 Sqrt[kl2] (kl1 + kl2) Sqrt[kr2] (-kl3 + kr2) (kr1 + kr2) (kl2 - kr3) π^2))
```