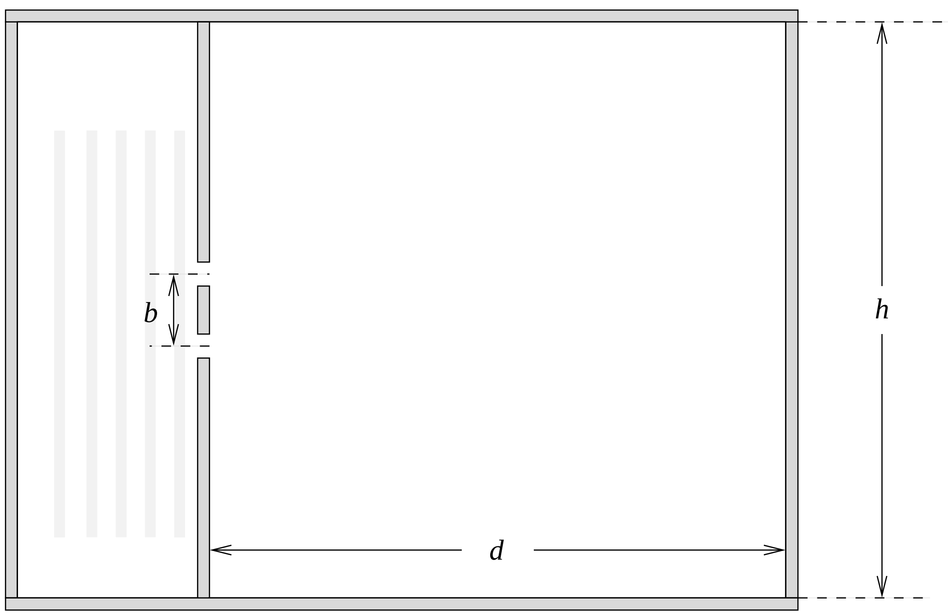
PHY 202 2002; Homework 12

Due Saturday May 3 at 5:00 PM at SE 227 or at 3510 5th Ave.

There will be a final physics tea on Thursday, May 1 between 8 and 10 PM at 3510 5th Ave. This Homework is relatively long and challenging, please get started right away and ask me lots of questions.

1. In a wave tank, plane waves—with wavelength $\lambda = 0.75\,\text{in}$—come in from the left and go through the two slits as shown.



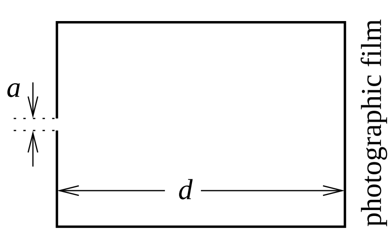
The dimensions are: $b = 1.875\,\text{in}$, $d = h = 15\,\text{in}$. In the following, ignore any reflections of waves off the sides of the wave tank.

(a) Find all the angles at which the outgoing waves will "add up."

(b) Find all the angles at which the outgoing wave will "cancel." (You will have to come up with a formula for this.)

(c) In the above picture, draw what you would expect to see in the right hand side of the wave tank. Your drawing should be accurate and detailed (use a ruler and/or protractor).

2. For the high school science fair, I design and build a pinhole camera whose depth is $d = 20\,\text{cm}$.



(a) If the width of the pinhole a is too large, the image of a far away object will be too blurry. Explain why using a picture and assuming light travels in straight "rays." Hint: what would the image of a far-away point source of light (a star, for example) look like?

(b) If a is too small, the image of a far-away point source of light will spread out inside the camera. Explain this effect using a picture.

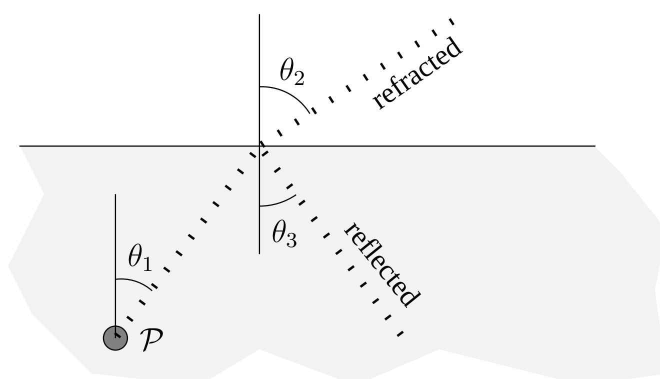
(c) Using your answer in 2a and 2b, find a formula for the size of the image of a far away point-source. Here, you can assume the angles are small: $\sin(\theta) \approx \theta$ and $\tan(\theta) \approx \theta$.

(d) The angular resolution ϕ is the minimum angle between two far-away point sources so that they produce distinct images on the photographic film. At what value of a will the camera have the best possible angular resolution?

(e) What is ϕ, in degrees, for yellow light?

Since the above estimates are rather rough, your answer will only be correct to within a factor of two.

3. Consider a light source $\mathcal{P}$ sitting in a bucket of water. A ray of light heading off at angle θ_1 hits the surface of the water; some of the light is refracted and some is reflected back into the water.



(a) What is θ_3?

(b) Use Snell's law to complete the table:

θ_1	θ_2
0°	
30°	
45°	
60°	

(c) Something goes wrong .... At exactly what angle does this happen? What happens to the light for angles larger than this?

Read about "total internal reflection" in your textbook.

4. There are two equations that relate the particle and wave properties of matter:

$$E = hf \quad \text{and} \quad p\lambda = h \ ,$$

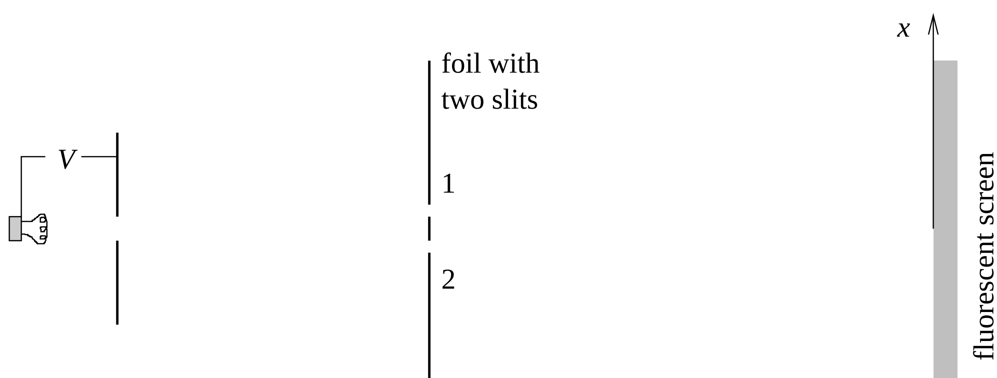
where E and p are the energy and momentum of the particle, f and λ are the frequency and wavelength of the corresponding wavefunction, and h is Planck's constant. The first relation is valid for massless particles (the photon) while the second can be applied to anything.

(a) What is the energy of one photon (in eV) from a HeNe laser, wavelength 632.8 nm.

(b) Violet light has a wavelength of 350 nm. What would the kinetic energy of the electrons have to be (in eV) if I were to make a electron beam with the same de Broglie wavelength?

(c) Use the above equations to derive a relation between the energy and momentum of a photon, without any λ or f. (For massive particles, this exercise will not work because the velocity of the wavefunction, the "phase velocity," does not match the velocity of the massive particle, the "group velocity.")

5.



Imagine that we made a pair of slits in a metal foil that are 0.13 μm apart (this is the current state-of-the-art in semiconductor fabrication). If we send 100 eV beam of electrons through these slits and they hit a fluorescent screen 1.0 m away, how far apart would the bright spots on the screen be?

6. Let us continue with the double-slit experiment for electrons. Just before an electron hits the fluorescent screen, the wavefunction of the electron that has traveled through slit 1 is

$$\psi_1(x,t) = C\,e^{\mathrm{i}kx - \mathrm{i}\omega t}$$

while the wavefunction of the electron that has traveled through slit 2 is

$$\psi_2(x,t) = C\,e^{-\mathrm{i}kx - \mathrm{i}\omega t}\ .$$

(This is a simplification. In real life, the wavefunction is more complicated.)

(a) If I cover up slit two, what is the intensity of the electrons hitting the screen as a function of x? (The intensity is proportional to the probability density for one electron).

(b) If I cover up slit one, what is the intensity of the electrons hitting the screen as a function of x?

(c) Now, consider the case where both slits are uncovered. What is the intensity of electrons hitting the screen as a function of x?

(d) Sketch a graph of intensity versus x.

(e) Based on your answer for problem 5, find a numerical value for k.

Hints: $e^{\mathrm{i}x} = \cos(x) + \mathrm{i}\sin(x)$, $e^{x+y} = e^x e^y$, and $|z| = \sqrt{z^* z}$, where z^* is the complex conjugate of z.

7. Before starting on this problem, read about "The Photoelectric Effect" in your textbook. Let us use the wave picture of light to calculate the minimum time needed to excite an electron enough so that it is ejected from a material. Consider a lamp emitting 40 watts of yellow light. A piece of Lithium is placed 10 cm away. The atoms in a Lithium crystal are about 0.15 nm apart. The "work function" of Lithium is 2.3 eV; the work function is the minimum energy needed to remove an electron from the surface.

   (a) What is the power of light falling on one Lithium atom on the surface of the crystal? Units: watts.

   (b) Assuming one electron in the Lithium atom absorbs all energy falling on that atom (a very optimistic assumption), how long would it take for the electron to have enough energy to jump off the Lithium crystal?

   (c) Compare your answer with the experimentally observed minimum time of less than 10^{-8} s.

   (d) Now, using $E = hf$, determine whether photons of yellow light have sufficient energy to knock electrons off the Lithium.

A sluggard does not plow in season;
so at harvest time he looks but finds nothing.
Prov. 20:4