PHY 202 Homework 3

Due Friday, February 6 at noon in my office.

The first test will be held on Friday, February 13.

1. Consider a uniform electric field in the $\hat{x}$-direction: $\mathbf{E} = E_x\hat{x}$. An electron is shot out of a gun in the $\hat{y}$ direction with initial velocity $\mathbf{v} = v_0\hat{y}$. Find an expression for the subsequent motion of the electron. Draw a graph showing the path of the electron; express the shape of the curve as a function y vs. x. In this problem, you can ignore the force of gravity.

2. Here is something we did in lecture. Consider a plane with charge density σ (units $\mathrm{C/m^2}$).

   (a) Find all of the symmetries of this system.

   (b) Use symmetry arguments to determine the direction of the resulting $\mathbf{E}$ field on each side of the plane.

   (c) Use symmetry arguements to determine how the $\mathbf{E}$ field behaves as a function of position.

   (d) Next you will use the Gauß law to find $\mathbf{E}$. Draw a picture illustrating an appropriate Gaußsche Oberfläche (Gaussian surface). A cube that contains part of the surface would be a good choice.

   (e) Using Gauß' law, determine $\mathbf{E}$ as a function of distance from the plane.

   This is more of an essay question than a calculation; be sure to explain your answer clearly.

3. Consider two parallel planes separated by a distance a. One plane has charge density σ and the other has charge density $-\sigma$.

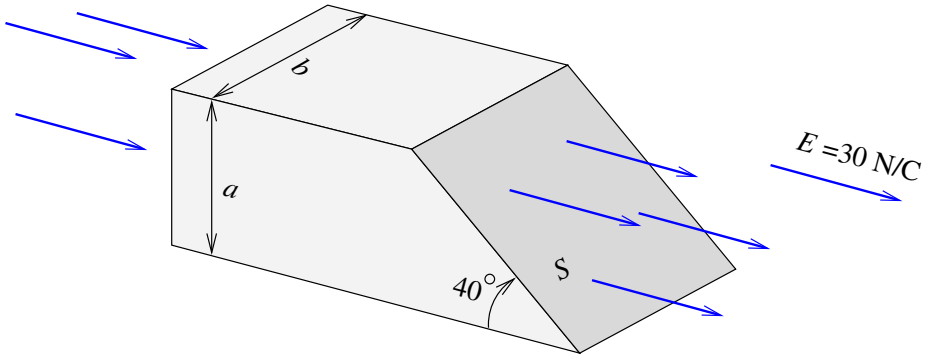
   (a) Discuss the symmetries of this system.

   (b) Determine $\mathbf{E}$ between the plates and outside of the plates u2sing the results from Problem 2 and the superposition principle.

   (c) What is the voltage difference between the plates as a function of σ and a? (Don't forget to include the contribution from both plates.)

4. A uniform electric field, magnitude $E = 30\,\mathrm{N/C}$, exists parallel to the axis of a rectangular block with sides of length $a = 3\,\mathrm{cm}$ and $b = 5\,\mathrm{cm}$. Let the surface S be the slanted side of the block. Calculate the surface integral $\int\int_S \mathbf{E}\cdot\hat{n}\,dA$ to

find the total electric flux Φ_E going through that surface.



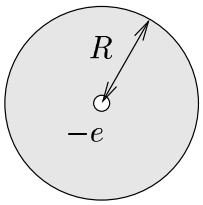
5. A very long metal rod, radius ρ, has a uniform surface charge density σ.

   (a) Find the symmetries of this system. What do these symmetries imply about the electric field?

   (b) Use Gauß' law to find the electric field at a radius 2ρ.

   (c) What is the electric field at a radius $\rho/2$? Explain your answer.

6. An early (incorrect) model of the hydrogen atom was suggested by J. J. Thompson. It consists of a positive cloud of charge $+e$ that is uniformly distributed throughout the volume of a sphere of radius R. The electron, charge $-e$, is located at the center of the sphere.



Charge $+e$ is distrubuted throughout the volume of the sphere with constant charge density

   (a) What are the symmetries of the positive cloud of charge $+e$? What does this imply about the $\mathbf{E}$ field generated by the positive cloud of charge?

   (b) Use Gauß' law to show that the electron $-e$ is at a *stable* equilibrium at the center.

   (c) If the electron is displaced x from the center, where $|x| < R$, show that it feels a force $F = -kx$. Show that $k = e^2/(4\pi\epsilon_0 R^3)$.

   (d) Recall that a particle that feels a force $F = -kx$ is a harmonic oscillator. If I displace the electron a small distance from the center, it will oscillate. What is the frequency of small amplitude oscillations of the electron?

   (e) If you look at the spectrum produced by hydrogen atoms, the brightest spectral line is the Lyman-α line with a frequency of $2.47 \cdot 10^{15}$ Hz. Find R such that the small amplitude oscillations will have this frequency. How does your value of R compare with the Bohr radius?

7. Consider a vacuum tube having two parallel plates separated by a distance of 5 cm. The voltage difference is 1000 V. An electron is emitted from the negatively charged plate with negligible kinetic energy.

   (a) What is the acceleration of the electron? (Define variables and your answer as an algebraic expression.)

   (b) What is the velocity of the electron, in m/s, just before it strikes the positively charged plate?

   (c) What is the electron's maximum kinetic energy? Express your answer in joules and in electron volts (eV).

   (d) How long does it take for the electron to traverse the region between the plates?

   (e) If the separation between the plates is doubled, how do your answers for (7a)–(7d) change?