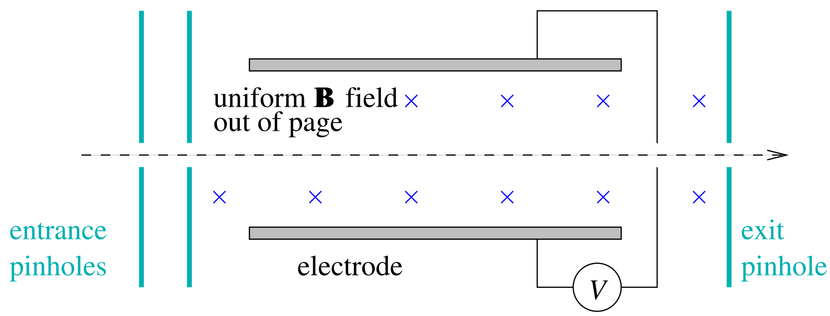
PHY 202 Homework 7

Due Friday, March 26 at 4:30 PM in my office.

1. A charged particle velocity filter is constructed from a region containing perpendicular electric and magnetic fields. If a charged particle moves through this region at just the right speed v the force of the magnetic field and force of the electric field cancel each other and the particle moves in a straight line.



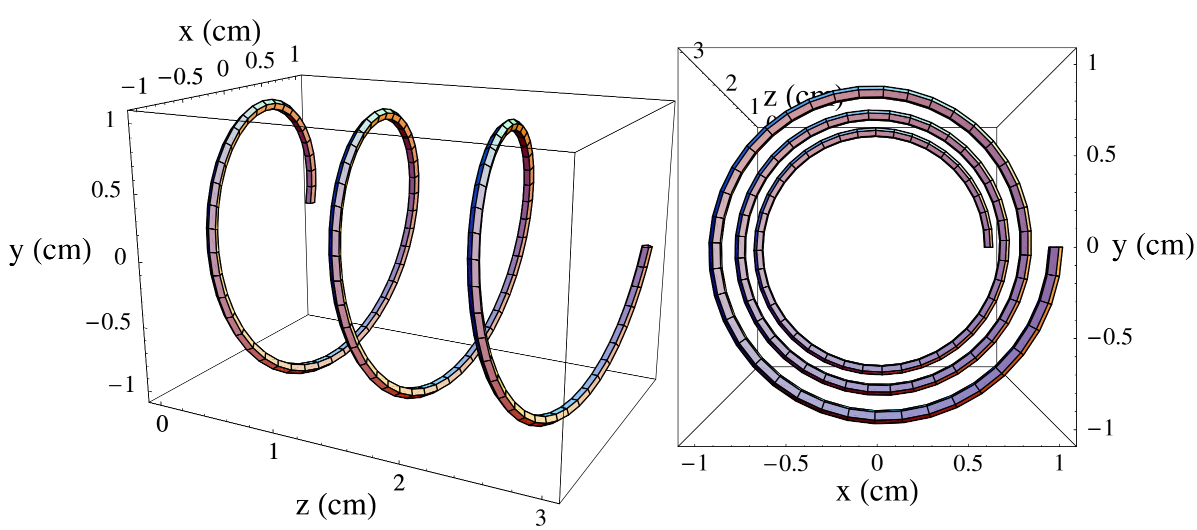
   Particles that are moving too fast—or too slow—will move in a curved path and hit the exit pinhole.

   (a) For a particle to move in a straight line, should the top electrode or the bottom electrode have positive charge?

   (b) For a given $||\mathbf{E}||$ and $||\mathbf{B}||$, what speed v is selected?

   (c) Choose values of $||\mathbf{E}||$ and $||\mathbf{B}||$ so that the velocity filter will select electrons with 1000 eV of kinetic energy.

2. A uniform magnetic field B exists parallel to the z-axis. An electron starts on the $z = 0$ plane and moves through the region as shown.



   The equation of a particle moving in a helix is:

$$\mathbf{x}(t) = r\cos(\omega t)\,\hat{x} + r\sin(\omega t)\,\hat{y} + v_z t\,\hat{z}\ .$$

(a) Verify that this equation is indeed a solution of $\mathbf{F} = m\mathbf{a}$.

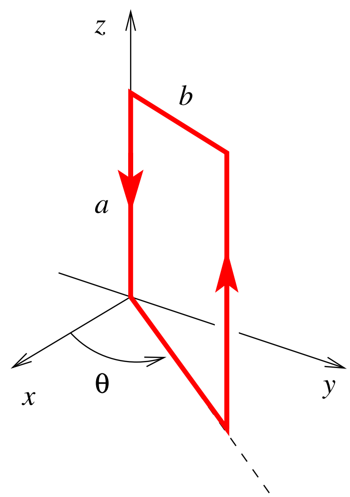
(b) Is the magnetic field in the positive or in the negative z-direction?

(c) From the picture, what is the radius r of the helix, in meters?

(d) What is the "pitch" of the helix, in meters? The pitch is the distance traveled in the z-direction during one rotation of the helix.

(e) (This part is pretty difficult) If it takes 1 μs for the particle to move through the above box, what is the *speed* of the particle in m/s? How big is B, in tesla?

3. Consider a rectangular loop, width b and height a, carrying current I in the direction shown.

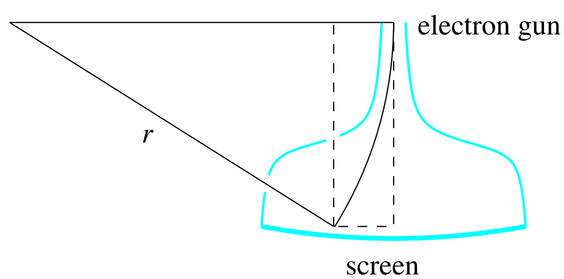


(a) Find the magnetic moment $\boldsymbol{\mu}$ expressed in terms of $\hat{x}$, $\hat{y}$, and $\hat{z}$.

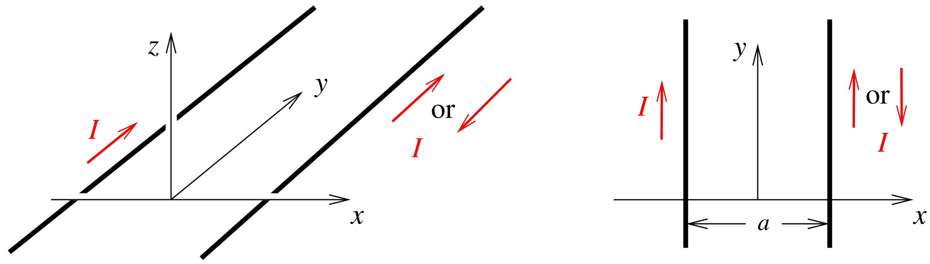
(b) If I apply a uniform magnetic field $\mathbf{B} = B_x\hat{x}$ to the loop, what is the torque exerted on the loop? (Find both magnitude and direction).

(c) What value of θ would correspond to the lowest potential energy? What θ corresponds to maximum potential energy? What is the difference in potential energy between these two.

4. Assume that electrons in the cathode ray tube (CRT) of an oscilloscope have an energy of 10 keV as they travel towards the screen. If the earth's magnetic field is about 0.6×10^{-4} T, how much can the earth's magnetic field deflect the electron beam before it hits the screen? Assume the length of the tube is about 20 cm.



5. Two infinitely long wires lie in the xy-plane parallel to each other, a distance $a = 10\,\text{cm}$ apart. A current of $I = 1\,\text{A}$ is flowing through each wire.



(a) First, let us consider *one* wire only. List the symmetries of this wire (using the appropriate coordinate system). Use Ampère's law to find the magnetic field produced by this wire at a distance r from the wire.

(b) Let us consider both wires and lets us assume that the currents of the wires are flowing in the *same* direction. Use the Lorentz force law to determine the force per unit length felt by each wire due to the other wire. Determine both the magnitude and the direction. Do the wires attract or repel?

(c) Use the superposition principle to find the magnetic field halfway between the two wires. Draw a picture of the magnetic field lines in the xz-plane.

(d) Finally, assume that the currents are flowing in *opposite* directions. What is the force per unit length felt by each wire due to the other wire. Determine both the magnitude and the direction. Do the wires attract or repel?

(e) What is the magnetic field halfway between the two wires? Draw a picture of the magnetic field lines in the xz-plane.

(f) Find an approximate form for the magnetic field in the $z = 0$ plane in the limit $x \gg a$.

A sluggard does not plow in season;
so at harvest time he looks but finds nothing.
Prov. 24:26