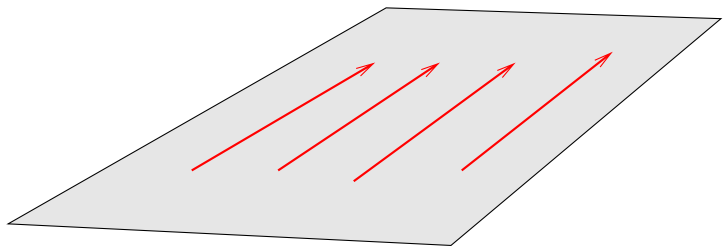
PHY 202 Homework 8

Due Thursday, April 8 at 12:20 PM outside my office.

1. Consider an infinitely large, thin sheet of metal carrying a uniform current per unit length λ.

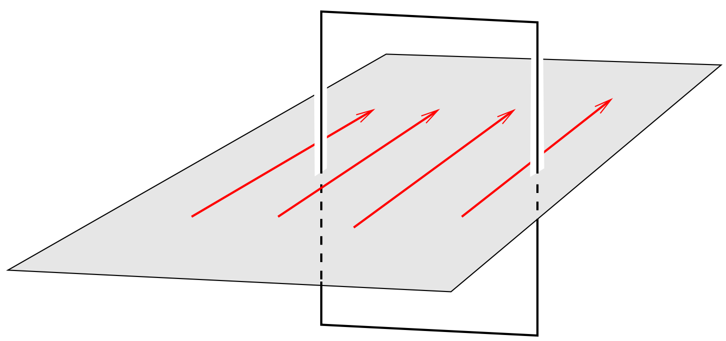


   (a) Draw a nice picture and define a coordinate system.

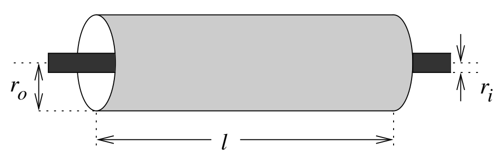
   (b) Find the symmetries of the current.

   (c) What do these symmetries and the "other" right hand rule imply about the magnetic field $\mathbf{B}$? Be specific.

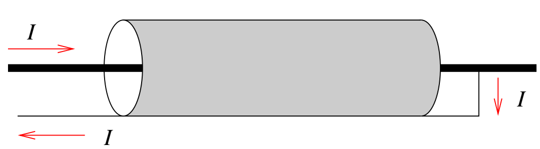
   (d) Use Ampère's law to find the magnetic field. Your closed path should be a rectangle enclosing some of the sheet:



2. Consider a coax cable of length ℓ, outer conductor radius r_o and inner conductor radius r_i.



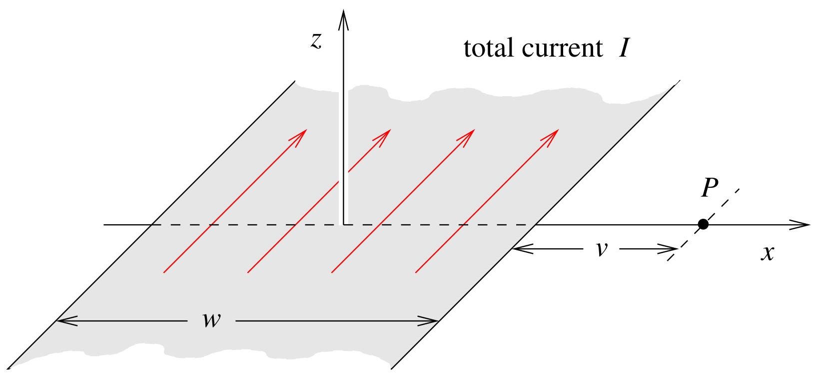
   A current I flows through the inner wire and returns on the outer wire.



   (a) Assuming the cable is very long (ignoring what happens at the ends) and that the current is uniformly distributed on the surface of each conductor, list the symmetries of the magnetic field. What is the direction of the magnetic field?

(b) Use Ampère's law to find the magnetic field both inside and outside the cable (for radii larger than r_i). Your answer should include a nice picture illustrating your integration paths.

3. Consider a long, thin strip of metal with width w carrying current I. Using the formula for the magnetic field of a straight wire, calculate the magnetic field $\mathbf{B}$ at a point P which lies in the same plane as the strip and is an distance v from the edge of the strip. Assume that the current is uniformly distributed across the strip.



(a) What are the symmetries of the current?

(b) What is the direction of $\mathbf{B}$ at P?

(c) What is the magnitude of $\mathbf{B}$ at P?

Use a calculus approach: consider the strip as the sum of many 'wires' and integrate over the magnetic field produced by each of these 'wires.'

4. Consider a very long solenoid of radius R and n turns per unit length (assume n is large). In class, we calculated the magnetic field produced by such a device. If I connect the coil to a battery, the current in the coil, as a function of time, will have the form

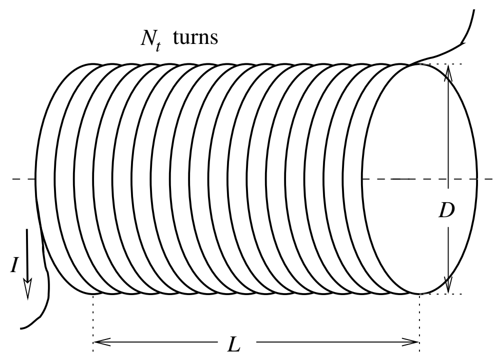
$$I(t) = I_f \left(1 - e^{-\alpha t}\right)$$

due to the finite resistance of the wire. (We will discuss this in class next week.)

(a) Find the derivative $\frac{d}{dt} I(t)$.

(b) Now, I put a small coil inside the large one. If the small coil has k turns and radius a, what is the induced voltage in the small coil?

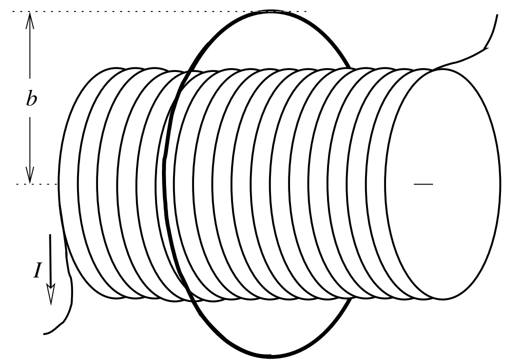
(c) In lab, we had the experiment with the toilet paper tube which we will approximate as an infinitely long solenoid. In that case, $N_t = 418$, $D = 4.7\,\text{cm}$, and $L = 9.2\,\text{cm}$. Also, the final current would be $I_f = 1.2\,\text{A}$, and the time constant is $\alpha = 8000\,1/\text{s}$. The parameters of the inner coil are: $k = 100$ turns and radius $a = 1.12\,\text{cm}$.



Find a numerical value for the induced voltage at time $t = 1\,\text{ms}$.

(d) Next, consider a coil, radius b and k turns, that is larger than the solenoid. Find the voltage induced in the coil by the solenoid. Express your answer in terms of $I'(t)$ and don't put any numerical values in. Note the following:

- The magnetic field exterior to the solenoid is zero, yet there is a nonzero induced voltage in the coil.
- The induced voltage is independent of b.



5. Use the equation for an inductor, $\mathcal{E} = -L\dfrac{di}{dt}$, to find the units of L (henries) in terms of the basic SI units: meter, coulomb, *et cetera.*

6. Beginning with the equations defining inductance L and resistance R, show that L/R has units of seconds.

7. Verify by substitution that the current $i(t) = \dfrac{V}{R}\left(1 - e^{-Rt/L}\right)$ is a solution of the equation $V - iR - L\dfrac{d}{dt}i = 0$.

8. A 9 V battery, a $50\,\Omega$ resistor, and a 10 H inductor are all connected in series. After the current in the circuit has equilibriated, find

   (a) the power dissapated in the resisitor,

   (b) the power consumed by the inductor, and

   (c) the total energy stored in the inductor.

If a man loudly blesses his neighbor early in the morning,
it will be taken as a curse.
Prov. 27:14